Homework 10.

Do as many as you can in class
Turn in at least Four (4).

SECTION 4.2 Summary

A **probability model** for a random phenomenon consists of a sample space S and an assignment of probabilities P.

The **sample space S** is the set of all possible outcomes of the random phenomenon. Sets of outcomes are called **events.** P assigns a number $P(A)$ to an event A as its probability.

The **complement A^c** of an event A consists of exactly the outcomes that are not in A. Events A and B are **disjoint** if they have no outcomes in common. Events A and B are **independent** if knowing that one event occurs does not change the probability we would assign to the other event.

Any assignment of probability must obey the rules that state the basic properties of probability:

Rule 1. $0 \leq P(A) \leq 1$ for any event A.
Rule 2. $P(S) = 1$.
Rule 3. Addition rule: If events A and B are **disjoint,** then $P(A \text{ or } B) = P(A) + P(B)$.
Rule 4. Complement rule: For any event A, $P(A^c) = 1 - P(A)$.
Rule 5. Multiplication rule: If events A and B are **independent,** then $P(A \text{ and } B) = P(A)P(B)$.

SECTION 4.2 Exercises

For Exercise 4.10, see page 234; for Exercise 4.11, see page 235; for Exercises 4.12 and 4.13, see page 238; for Exercises 4.14 and 4.15, see page 239; for Exercise 4.16, see page 241; for Exercise 4.17, see page 242; and for Exercise 4.18, see page 242.

4.19 What's wrong? In each of the following scenarios, there is something wrong. Describe what is wrong and give a reason for your answer.

(a) If two events are disjoint, we can multiply their probabilities to determine the probability that they will both occur.

(b) If the probability of A is 0.6 and the probability of B is 0.5, the probability of both A and B happening is 1.1.

(c) If the probability of A is 0.35, then the probability of the complement of A is -0.35.

4.20 What's wrong? In each of the following scenarios, there is something wrong. Describe what is wrong and give a reason for your answer.

(a) If the sample space consists of two outcomes, then each outcome has probability 0.5.

(b) If we select a digit at random, then the probability of selecting a 2 is 0.2.

(c) If the probability of A is 0.2, the probability of B is 0.3, and the probability of A and B is 0.5, then A and B are independent.

4.21 Evaluating Web page designs. You are a Web page designer and you set up a page with five different links. A user of the page can click on one of the links or he or she can leave that page. Describe the sample space for the outcome of a visitor to your Web page.

4.22 Record the length of time spent on the page. Refer to the previous exercise. You also decide to measure the length of time a visitor spends on your page. Give the sample space for this measure.

4.23 Ringtones. What are the popular ringtones? The web site `funtonia.com` updates its list of top ringtones frequently. Here are probabilities for the top 10 ringtones listed by the site recently:[7]

Ringtone	Probability	Ringtone	Probability
Empire State of Mind	0.180	Bad Romance	0.081
Baby By Me	0.136	I Can Transform Ya	0.075
Forever	0.114	Down	0.070
Party in the USA	0.107	I Gotta Feeling	0.068
Fireflies	0.103	Money To Blow	0.066

(a) What is the probability that a randomly selected ringtone from this list is either Empire State of Mind or I Gotta Feeling?

(b) What is the probability that a randomly selected ringtone from this list is not Empire State of Mind and not I Gotta Feeling? Be sure to show how you computed this answer.

4.24 More ringtones. Refer to the previous exercise.

(a) If two ringtones are selected independently, what is the probability that both are Party in the USA?

(b) Describe in words the complement of the event described in part (a) of this exercise. Find the probability of this event.

4.25 Distribution of blood types. All human blood can be "ABO-typed" as one of O, A, B, or AB, but the distribution of the types varies a bit among groups of people. Here is the distribution of blood types for a randomly chosen person in the United States:[8]

Blood type	A	B	AB	O
U.S. probability	0.42	0.11	?	0.44

(a) What is the probability of type AB blood in the United States?

(b) Maria has type B blood. She can safely receive blood transfusions from people with blood types O and B. What is the probability that a randomly chosen person from the United States can donate blood to Maria?

4.26 Blood types in Ireland. The distribution of blood types in Ireland differs from the U.S. distribution given in the previous exercise:

Blood type	A	B	AB	O
Ireland probability	0.35	0.10	0.03	0.52

Choose a person from the United States and a person from Ireland at random, independently of each other. What is the probability that both have type O blood? What is the probability that both have the same blood type?

4.27 Are the probabilities legitimate? In each of the following situations, state whether or not the given assignment of probabilities to individual outcomes is legitimate, that is, satisfies the rules of probability. If not, give specific reasons for your answer.

(a) Choose a college student at random and record gender and enrollment status: $P(\text{female full-time}) = 0.44$, $P(\text{female part-time}) = 0.56$, $P(\text{male full-time}) = 0.46$, $P(\text{male part-time}) = 0.54$.

(b) Deal a card from a shuffled deck: $P(\text{clubs}) = 16/52$, $P(\text{diamonds}) = 12/52$, $P(\text{hearts}) = 12/52$, $P(\text{spades}) = 12/52$.

(c) Roll a die and record the count of spots on the up-face: $P(1) = 1/3$, $P(2) = 0$, $P(3) = 1/6$, $P(4) = 1/3$, $P(5) = 1/6$, $P(6) = 0$.

4.28 French and English in Canada. Canada has two official languages, English and French. Choose a Canadian at random and ask, "What is your mother tongue?" Here is the distribution of responses, combining many separate languages from the broad Asian/Pacific region:[9]

Language	English	French	Asian/Pacific	Other
Probability	0.59	?	0.07	0.11

(a) What probability should replace "?" in the distribution?

(b) What is the probability that a Canadian's mother tongue is not English? Explain how you computed your answer.

4.29 Education levels of young adults. Choose a young adult (age 25 to 34 years) at random. The probability is 0.12 that the person chosen did not complete high school, 0.31 that the person has a high school diploma but no further education, and 0.29 that the person has at least a bachelor's degree.

(a) What must be the probability that a randomly chosen young adult has some education beyond high school but does not have a bachelor's degree?

(b) What is the probability that a randomly chosen young adult has at least a high school education?

4.30 Loaded dice. There are many ways to produce crooked dice. To *load* a die so that 6 comes up too often and 1 (which is opposite 6) comes up too seldom, add a bit of lead to the filling of the spot on the 1 face. Because the spot is solid plastic, this works even with transparent dice. If a die is loaded so that 6 comes up with probability 0.21 and the probabilities of the 2, 3, 4, and 5 faces are not affected, what is the assignment of probabilities to the six faces?

4.31 Rh blood types. Human blood is typed as O, A, B, or AB and also as Rh-positive or Rh-negative. ABO type and Rh-factor type are independent because they are governed by different genes. In the American population, 84% of people are Rh-positive. Use the information about ABO type in Exercise 4.25 to give the probability distribution of blood type (ABO and Rh) for a randomly chosen person.

4.32 Roulette. A roulette wheel has 38 slots, numbered 0, 00, and 1 to 36. The slots 0 and 00 are colored green, 18 of the others are red, and 18 are black. The dealer spins the wheel and at the same time rolls a small ball along the wheel in the opposite direction. The wheel is carefully balanced so that the ball is equally likely to land in any slot when the wheel slows. Gamblers can bet on various combinations of numbers and colors.

(a) What is the probability that the ball will land in any one slot?

(b) If you bet on "red," you win if the ball lands in a red slot. What is the probability of winning?

(c) The slot numbers are laid out on a board on which gamblers place their bets. One column of numbers on the board contains all multiples of 3, that is, 3, 6, 9, ..., 36. You place a "column bet" that wins if any of these numbers comes up. What is your probability of winning?

4.33 Winning the lottery. A state lottery's Pick 3 game asks players to choose a three digit number, 000 to 999. The state chooses the winning three-digit number at random, so that each number has probability 1/1000. You win if the winning number contains the digits in your number, in any order.

(a) Your number is 491. What is your probability of winning?

(b) Your number is 222. What is your probability of winning?

4.34 PINs. The personal identification numbers (PINs) for automatic teller machines usually consist of four digits. You notice that most of your PINs have at least one 0, and you wonder if the issuers use lots of 0s to make the numbers easy to remember. Suppose that PINs are assigned at random, so that all four-digit numbers are equally likely.

(a) How many possible PINs are there?

(b) What is the probability that a PIN assigned at random has at least one 0?

4.35 Universal blood donors. People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7% of the American population have O-negative blood. If 10 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor?

4.36 Disappearing Internet sites. Internet sites often vanish or move, so that references to them can't be followed. In fact, 13% of Internet sites referenced in papers in major scientific journals are lost within two years after publication.[10] If a paper contains seven Internet references, what is the probability that all seven are still good two years later? What specific assumptions did you make in order to calculate this probability?

4.37 Is this calculation correct? Government data show that 6% of the American population are at least 75 years of age and that about 51% are women. Explain why it is wrong to conclude that because $(0.06)(0.51) = 0.0306$ about 3% of the population are women aged 75 or over.

4.38 Colored dice. Here's more evidence that our intuition about chance behavior is not very accurate. A six-sided die has four green and two red faces, all equally probable. Psychologists asked students to say which of these color sequences is most likely to come up at the beginning of a long set of rolls of this die:

RGRRR

RGRRRG

GRRRRR

More than 60% chose the second sequence.[11] What is the correct probability of each sequence?

4.39 Random walks and stock prices. The "random walk" theory of securities prices holds that price movements in disjoint time periods are independent of each other. Suppose that we record only whether the price is up or down each year, and that the probability that our portfolio rises in price in any one year is 0.65. (This probability is approximately correct for a portfolio containing equal dollar amounts of all common stocks listed on the New York Stock Exchange.)

(a) What is the probability that our portfolio goes up for three consecutive years?

(b) If you know that the portfolio has risen in price two years in a row, what probability do you assign to the event that it will go down next year?

(c) What is the probability that the portfolio's value moves in the same direction in both of the next two years?

4.40 CHALLENGE **Axioms of probability.** Show that any assignment of probabilities to events that obeys Rules 2 and 3 on page 236 automatically obeys the complement rule (Rule 4). This implies that a mathematical treatment of probability can start from just Rules 1, 2, and 3. These rules are sometimes called *axioms* of probability.

4.41 CHALLENGE **Independence of complements.** Show that if events A and B obey the multiplication rule, $P(A \text{ and } B) = P(A)P(B)$, then A and the complement B^c of B also obey the multiplication rule, $P(A \text{ and } B^c) = P(A)P(B^c)$. That is, if events A and B are independent, then A and B^c are also independent. (*Hint:* Start by drawing a Venn diagram and noticing that the events "A and B" and "A and B^c" are disjoint.)

Mendelian inheritance. *Some traits of plants and animals depend on inheritance of a single gene. This is called Mendelian inheritance, after Gregor Mendel (1822--1884). Exercises 4.42 to 4.45 are based on the following information about Mendelian inheritance of blood type.*

Each of us has an ABO blood type, which describes whether two characteristics called A and B are present. Every human being has two blood type alleles (gene forms), one inherited from our mother and one from our father. Each of these alleles can be A, B, or O. Which two we inherit determines our blood type. Here is a table that shows what our blood type is for each combination of two alleles:

Alleles inherited	Blood type
A and A	A
A and B	AB
A and O	A
B and B	B
B and O	B
O and O	O

We inherit each of a parent's two alleles with probability 0.5. We inherit independently from our mother and father.

4.42 Blood types of children. Hannah and Jacob both have alleles A and B.

(a) What blood types can their children have?

(b) What is the probability that their next child has each of these blood types?

4.43 Parents with alleles B and O. Nancy and David both have alleles B and O.

(a) What blood types can their children have?

(b) What is the probability that their next child has each of these blood types?

4.44 Two children. Jennifer has alleles A and O. José has alleles A and B. They have two children. What is the probability that both children have blood type A? What is the probability that both children have the same blood type?

4.45 Three children. Jasmine has alleles A and O. Joshua has alleles B and O.

(a) What is the probability that a child of these parents has blood type O?

(b) If Jasmine and Joshua have three children, what is the probability that all three have blood type O? What is the probability that the first child has blood type O and the next two do not?

4.9. The true probability (assuming perfectly fair dice) is $1 - \left(\frac{5}{6}\right)^4 \doteq 0.5177$, so students should conclude that the probability is "quite close to 0.5."

4.10. Sample spaces will likely include blonde, brunette (or brown), black, red, and gray. Depending on student imagination (and use of hair dye), other colors may be listed; there should at least be options to answer "other" and "bald."

4.11. One possibility: from 0 to __ hours (the largest number should be big enough to include all possible responses). In addition, some students might respond with fractional answers (e.g., 3.5 hours).

4.12. $P(\text{Black or White}) = 0.07 + 0.02 = 0.09$.

4.13. $P(\text{Blue, Green, Black, Brown, Grey, or White}) = 1 - P(\text{Purple, Red, Orange, or Yellow}) = 1 - (0.14 + 0.08 + 0.05 + 0.03) = 1 - 0.3 = 0.7$. Using Rule 4 (the complement rule) is slightly easier, because we only need to add the four probabilities of the colors we do *not* want, rather than adding the six probabilities of the colors we want.

4.14. $P(\text{not } 1) = 1 - 0.301 = 0.699$.

4.15. In Example 4.13, $P(B) = P(6 \text{ or greater})$ was found to be 0.222, so $P(A \text{ or } B) = P(A) + P(B) = 0.301 + 0.222 = 0.523$.

4.16. For each possible value $(1, 2, \ldots, 6)$, the probability is $1/6$.

4.17. If T_k is the event "get tails on the kth flip," then T_1 and T_2 are independent, and $P(\text{two tails}) = P(T_1 \text{ and } T_2) = P(T_1)P(T_2) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

4.18. If A_k is the event "the kth card drawn is an ace," then A_1 and A_2 are *not* independent; in particular, if we know that A_1 occurred, then the probability of A_2 is only $\frac{3}{51}$.

4.19. **(a)** The probability that both of two disjoint events occur is 0. (Multiplication is appropriate for *independent* events.) **(b)** Probabilities must be no more than 1; $P(A \text{ and } B)$ will be no more than 0.5. (We cannot determine this probability exactly from the given information.) **(c)** $P(A^c) = 1 - 0.35 = 0.65$.

4.20. **(a)** The two outcomes (say, A and B) in the sample space need not be equally likely. The only requirements are that $P(A) \geq 0$, $P(B) \geq 0$, and $P(A) + P(B) = 1$. **(b)** In a table of random digits, each digit has probability 0.1. **(c)** If A and B were independent, then $P(A \text{ and } B)$ would equal $P(A)P(B) = 0.06$. (That is, probabilities are multiplied, not added.) In fact, the given probabilities are impossible, because $P(A \text{ and } B)$ must be less than the smaller of $P(A)$ and $P(B)$.

4.21. There are six possible outcomes: { link1, link2, link3, link4, link5, leave }.

4.22. There are an infinite number of possible outcomes, and the description of the sample space will depend on whether the time is measured to any degree of accuracy (S is the set of all positive numbers) or rounded to (say) the nearest second ($S = \{0, 1, 2, 3, \ldots\}$), or nearest tenth of a second ($S = \{0, 0.1, 0.2, 0.3 \ldots\}$).

4.23. **(a)** P("Empire State of Mind" or "I Gotta Feeling") $= 0.180 + 0.068 = 0.248$.
(b) P(neither "Empire State of Mind" nor "I Gotta Feeling") $= 1 - 0.248 = 0.752$.

4.24. **(a)** If R_k is the event "'Party in the USA' is the kth chosen ringtone," then $P(R_1 \text{ and } R_2) = P(R_1)P(R_2) = 0.107^2 = 0.011449$. **(b)** The complement would be "at most one ringtone is 'Party in the USA.'" $P[(R_1 \text{ and } R_2)^c] = 1 - P(R_1 \text{ and } R_2) = 0.988551$.

4.25. **(a)** The given probabilities have sum 0.97, so P(type AB) $= 0.03$.
(b) P(type O or B) $= 0.44 + 0.11 = 0.55$.

4.26. P(both are type O) $= (0.44)(0.52) = 0.2288$; P(both are the same type) $=$ $(0.42)(0.35) + (0.11)(0.10) + (0.03)(0.03) + (0.44)(0.52) = 0.3877$.

4.27. **(a)** Not legitimate because the probabilities sum to 2. **(b)** Legitimate (for a nonstandard deck). **(c)** Legitimate (for a nonstandard die).

4.28. **(a)** The given probabilities have sum 0.77, so P(French) $= 0.23$.
(b) P(not English) $= 1 - 0.59 = 0.41$, using Rule 4. (Or, add the other three probabilities.)

4.29. **(a)** The given probabilities have sum 0.72, so this probability must be 0.28.
(b) P(at least a high school education) $= 1 - P$(has not finished HS) $= 1 - 0.12 = 0.88$. (Or add the other three probabilities.)

4.30. The probabilities of 2, 3, 4, and 5 are unchanged (1/6), so P(⚀ or ⚅) must still be 1/3. If P(⚅) $= 0.21$, then P(⚀) $= \frac{1}{3} - 0.21 = 0.12\overline{3}$ (or $\frac{37}{300}$). The complete table follows.

Face	⚀	⚁	⚂	⚃	⚄	⚅
Probability	$0.12\overline{3}$	1/6	1/6	1/6	1/6	0.21

4.31. For example, the probability for A-positive blood is $(0.42)(0.84) = 0.3528$ and for A-negative $(0.42)(0.16) = 0.0672$.

Blood type	A+	A–	B+	B–	AB+	AB–	O+	O–
Probability	0.3528	0.0672	0.0924	0.0176	0.0252	0.0048	0.3696	0.0704

4.32. **(a)** All are equally likely; the probability is 1/38. **(b)** Because 18 slots are red, the probability of a red is $P(\text{red}) = \frac{18}{38} \doteq 0.474$. **(c)** There are 12 winning slots, so P(win a column bet) $= \frac{12}{38} \doteq 0.316$.

4.33. **(a)** There are six arrangements of the digits 4, 9, and 1 (491, 419, 941, 914, 149, 194), so that $P(\text{win}) = \frac{6}{1000} = 0.006$. **(b)** The only winning arrangement is 222, so $P(\text{win}) = \frac{1}{1000} = 0.001$.

4.34. **(a)** There are $10^4 = 10{,}000$ possible PINs (0000 through 9999).* **(b)** The probability that a PIN has *no* 0s is 0.9^4 (because there are 9^4 PINs that can be made from the nine nonzero digits), so the probability of at least one 0 is $1 - 0.9^4 = 0.3439$.

*If we assume that PINs cannot have leading 0s, then there are only 9000 possible codes (1000–9999), and the probability of at least one 0 is $1 - \frac{9^4}{9000} = 0.271$.

4.35. $P(\text{none are O-negative}) = (1 - 0.07)^{10} \doteq 0.4840$, so $P(\text{at least one is O-negative}) \doteq 1 - 0.4840 = 0.5160$.

4.36. If we assume that each site is independent of the others (and that they can be considered as a random sample from the collection of sites referenced in scientific journals), then $P(\text{all seven are still good}) = 0.87^7 \doteq 0.3773$.

4.37. This computation would only be correct if the events "a randomly selected person is at least 75" and "a randomly selected person is a woman" were independent. This is likely not true; in particular, as women have a greater life expectancy than men, this fraction is probably greater than 3%.

4.38. As $P(\text{R}) = \frac{2}{6}$ and $P(\text{G}) = \frac{4}{6}$, and successive rolls are independent, the respective probabilities are:

$$\left(\tfrac{2}{6}\right)^4\left(\tfrac{4}{6}\right) = \tfrac{2}{243} \doteq 0.00823,\ \left(\tfrac{2}{6}\right)^4\left(\tfrac{4}{6}\right)^2 = \tfrac{4}{729} \doteq 0.00549, \text{ and } \left(\tfrac{2}{6}\right)^5\left(\tfrac{4}{6}\right) = \tfrac{2}{729} \doteq 0.00274.$$

4.39. **(a)** $(0.65)^3 \doteq 0.2746$ (under the random walk theory). **(b)** 0.35 (because performance in separate years is independent). **(c)** $(0.65)^2 + (0.35)^2 = 0.545$.

4.40. For any event A, along with its complement A^c, we have $P(S) = P(A \text{ or } A^c)$ because "A or A^c" includes all possible outcomes (that is, it is the entire sample space S). By Rule 2, $P(S) = 1$, and by Rule 3, $P(A \text{ or } A^c) = P(A) + P(A^c)$, because A and A^c are disjoint. Therefore, $P(A) + P(A^c) = 1$, from which Rule 4 follows.

4.41. Note that $A = (A \text{ and } B) \text{ or } (A \text{ and } B^c)$, and the events $(A \text{ and } B)$ and $(A \text{ and } B^c)$ are disjoint, so Rule 3 says that

$$P(A) = P\big((A \text{ and } B) \text{ or } (A \text{ and } B^c)\big) = P(A \text{ and } B) + P(A \text{ and } B^c).$$

If $P(A \text{ and } B) = P(A)P(B)$, then we have $P(A \text{ and } B^c) = P(A) - P(A)P(B) = P(A)(1 - P(B))$, which equals $P(A)P(B^c)$ by the complement rule.

4.42. **(a)** Hannah and Jacob's children can have alleles AA, BB, or AB, so they can have blood type A, B, or AB. (The table on the right shows the possible combinations.) **(b)** Either note that the four combinations in the table are equally likely, or compute:

	A	B
A	AA	AB
B	AB	BB

$$P(\text{type A}) = P(\text{A from Hannah and A from Jacob}) = P(\text{A}_H)P(\text{A}_J) = 0.5^2 = 0.25$$
$$P(\text{type B}) = P(\text{B from Hannah and B from Jacob}) = P(\text{B}_H)P(\text{B}_J) = 0.5^2 = 0.25$$
$$P(\text{type AB}) = P(\text{A}_H)P(\text{B}_J) + P(\text{B}_H)P(\text{A}_J) = 2 \cdot 0.25 = 0.5$$

4.43. **(a)** Nancy and David's children can have alleles BB, BO, or OO, so they can have blood type B or O. (The table on the right shows the possible combinations.) **(b)** Either note that the four combinations in the table are equally likely or compute $P(\text{type O}) = P(\text{O from Nancy and O from David}) = 0.5^2 = 0.25$ and $P(\text{type B}) = 1 - P(\text{type O}) = 0.75$.

	B	O
B	BB	BO
O	BO	OO

4.44. Any child of Jennifer and José has a 50% chance of being type A (alleles AA or AO), and each child inherits alleles independently of other children, so $P(\text{both are type A}) = 0.5^2 = 0.25$. For one child, we have $P(\text{type A}) = 0.5$ and $P(\text{type AB}) = P(\text{type B}) = 0.25$, so that $P(\text{both have same type}) = 0.5^2 + 0.25^2 + 0.25^2 = 0.375 = \frac{3}{8}$.

	A	O
A	AA	AO
B	AB	BO

4.45. **(a)** Any child of Jasmine and Joshua has an equal (1/4) chance of having blood type AB, A, B, or O (see the allele combinations in the table). Therefore, $P(\text{type O}) = 0.25$. **(b)** $P(\text{all three have type O}) = 0.25^3 = 0.015625 = \frac{1}{64}$. $P(\text{first has type O, next two do not}) = 0.25 \cdot 0.75^2 = 0.140625 = \frac{9}{64}$.

	A	O
B	AB	BO
O	AO	OO

4.46. $P(\text{grade of D or F}) = P(X = 0 \text{ or } X = 1) = 0.05 + 0.04 = 0.09$.

4.47. If H is the number of heads, then the distribution of H is as given on the right. $P(H = 0)$, the probability of two tails was previously computed in Exercise 4.17.

Value of H	0	1	2
Probabilities	1/4	1/2	1/4

4.48. $P(0.1 < X < 0.4) = 0.3$.

4.49. **(a)** The probabilities for a discrete *random variable* always add to one. **(b)** Continuous random variables can take values from any interval, not just 0 to 1. **(c)** A Normal random variable is continuous. (Also, a distribution is *associated with* a random variable, but "distribution" and "random variable" are not the same things.)