§1.2

(No Monday)

Math 151-2015XB-W2 Tues

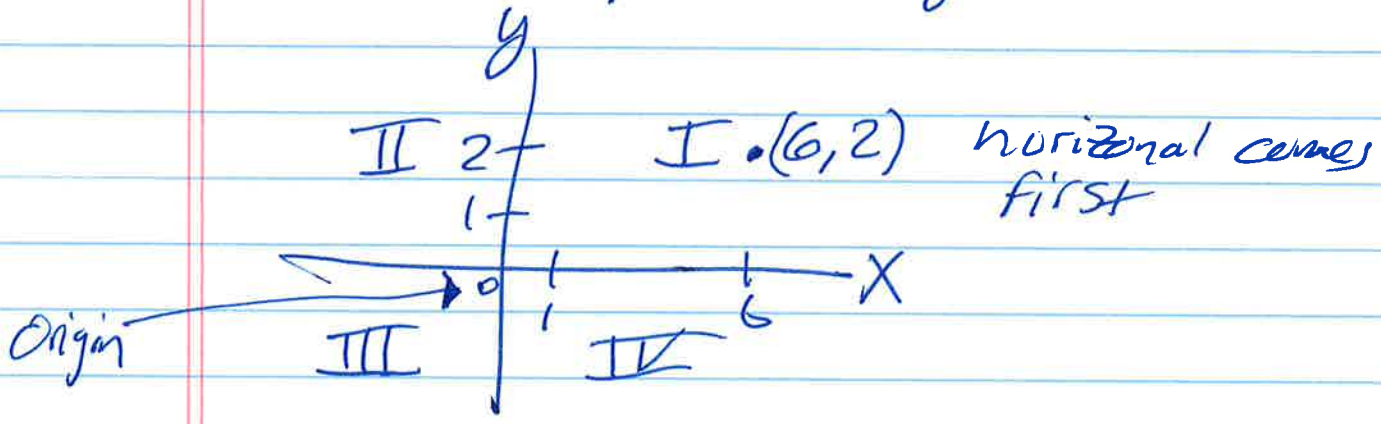
§1.2 Graphs and Lines

Cartesian Coordinate System

Two number lines
- horizontal axis (x axis) traditionally
- vertical axis (y axis)

} together the coordinate axes

Axes divide plane into quadrants



1st coordinate - abscissa
2nd ordinate

(0, 0) origin

1-1 Correspondence goes both ways

- For every point in plane there is a unique pair of numbers (x, y) the coordinates of the point
- For every pair of number (x, y) there is a unique point in plane

* You can "graph" a set of points
* This can be a solution set of an equation or inequality.
* We have done this already

$x \geq 5$

* But in the cartesian plane we have two axes, This is good for graphing solutions that have two variables x and y

* A linear equation in two variables is an equation that can be written in the standard form

$Ax + By = C$ A and B not both 0

A Solution of an equation in two variables is an ordered pair (x, y) that satisfies the equation

The solution set of the equation/inequality is the set of all solutions. Also called the graph of the equation/inequality

Linear equations have solution sets that are lines (thing the name). linear

Any line in the cartesian plane is the graph of a linear equation ie an equation of form $Ax + By = C$ A,B not both 0

If $A \neq 0$ and $B \neq 0$ then
$Ax + By = C$
can be written (solve for y

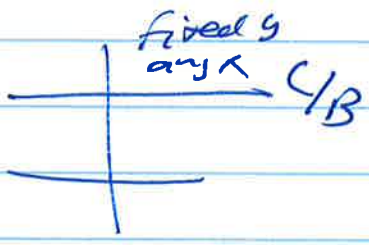
$By = -Ax + C$
$y = -A/B\,x + C/B$

$-A/B$ sometimes written m. $m = -A/B$

C/B sometimes written b. $b = C/B$

Equation written $y = mx + b$

If $A = 0$ and $B \neq 0$



$Ax + By = C \Rightarrow By = C$
$y = C/B$ this is a horizontal line

If $A \neq 0$ and $B = 0$

$Ax + By = C \Rightarrow Ax = C$
$x = C/A$ vertical line.

intercepts The y intercept (often written b) is the y coordinate of the point where the graph crosses the y-axis. To find, let $x=0$ and solve for y.

Eg $3x-4y=12$

Set $x=0$ $\quad -4y=12$

$y=-3$

$b=-3$

or graph passes through $(0,-3)$

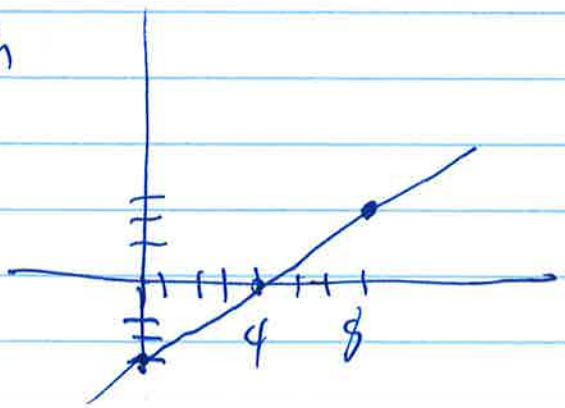
Likewise the x intercept (often written a) or is the x-coordinate of the point where the graph crosses the x-axis. To find set $y=0$ and solve for x

Eg $3x-4y=12$

$y=0$ $\quad 3x=12 \Rightarrow x=4$

graph passes through $(4,0)$

Graph



Try another point

$x=8$ $\quad 24-4y=12$

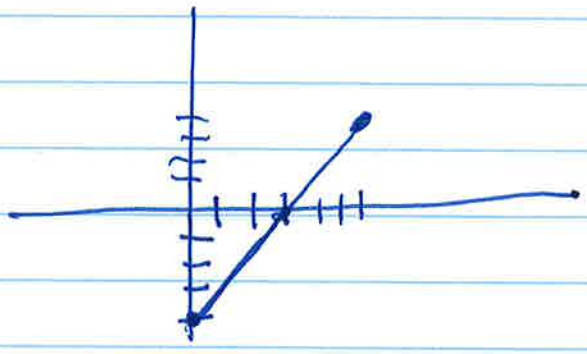
$y=3$

How would you find to try 8?

Graph $4x - 3y = 12$

$y = 0 \quad x = 3$

$x = 0 \quad y = -4$



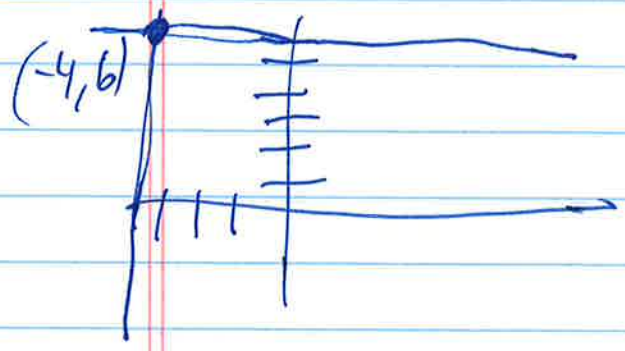
Check $x = 6 \quad 24 - 3y = 12$

$-3y = -12$

$y = 4$

Example

(A) Graph $x=-4$ and $y=6$ simultaneously in the same rectangular coordinate system



B) Write the equations of the vertical and horizontal lines that pass through

$(7,-5)$

$x=7$

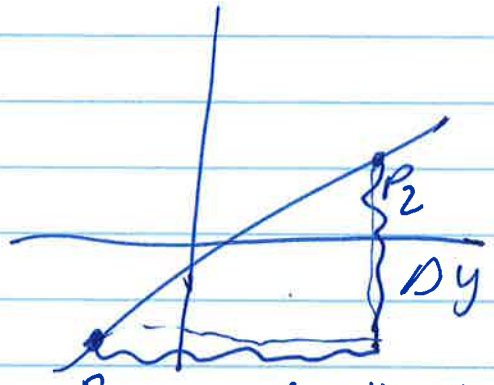
$y=-5$

Matched

(A) Graph $x=5$, $y=-3$ simultaneously

(B) Write equations for the vertical and horizontal lines that pass through $(-8,2)$

Slope - ratio of change in y to change in x, as the point on the line moves from 1 point P_1 to another P_2



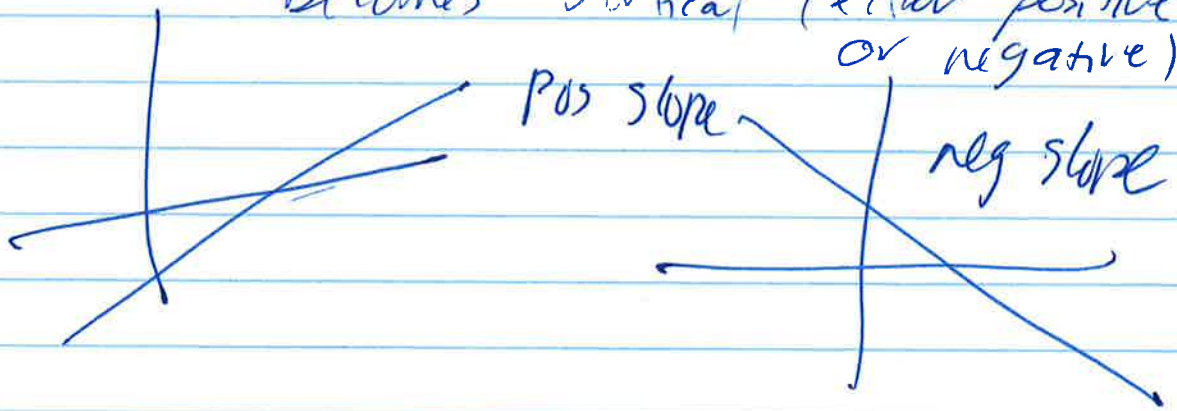
P_1 and P_2 can be any two points on line in any order. (Property of line)

$$\text{Slope} = \text{rise}/\text{run}$$

measure of steepness

horizontal line - slope 0

vertical line - slope undefined "∞"
gets very large as line becomes vertical (either positive or negative)



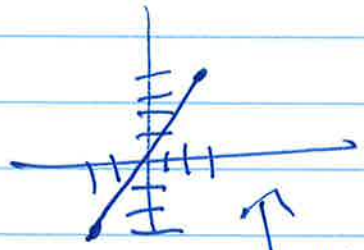
pg 8

Sketch a line through the pair of points

$(-3, -2)$ $(3, 4)$

Find slope

$\frac{4-(-2)}{3-(-3)} = \frac{6}{6} = 1$



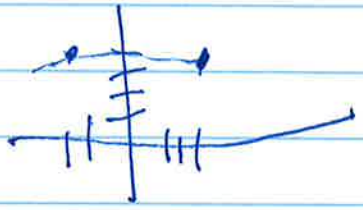
this is rough but it tells you slope is positive going up hill from left to right

Matched

Find slope between pairs of points

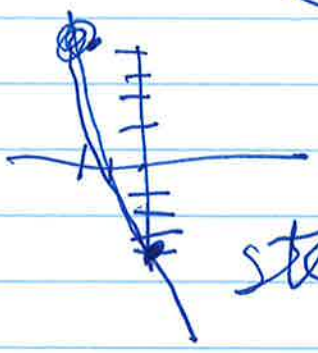
$(-2, 4), (3, 4)$

$\frac{4-4}{3-(-2)} = \frac{0}{5} = 0$



(B) $(-2, 4)$ $(0, -4)$

$$\frac{-4-4}{0+2} = \frac{-8}{2} = -4$$

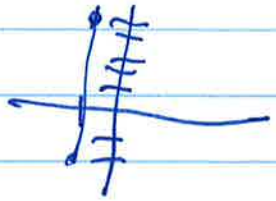


steep negative

(C) $(-1, 5)$ $(-1, -2)$

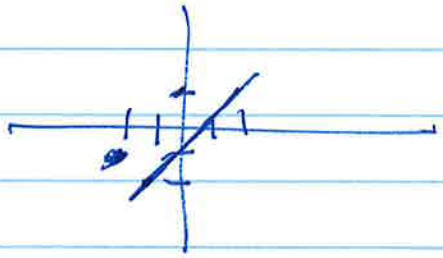
$$\frac{-2-5}{-1+1} = \frac{-7}{0} = \text{undefined}$$

vertical



(D) $(-1, -2)$ $(2, 1)$

$$\frac{1-(-2)}{2-(-1)} = \frac{3}{3} = 1$$



Let m be the slope of the line and b be the y intercept.

Then the equation of line is

$$y = mx + b$$

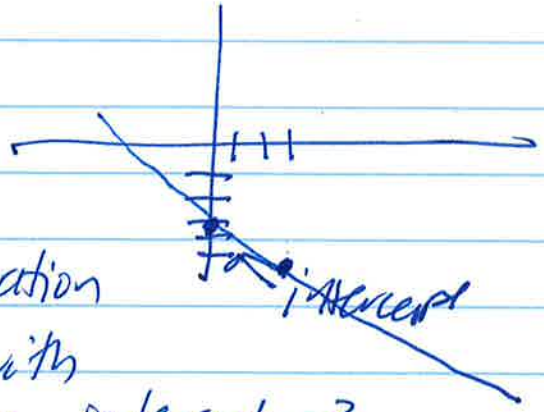
Called the slope-intercept form

(A) Find slope and intercept and graph

$$y = -\frac{2}{3}x - 3$$

slope $-\frac{2}{3}$ intercept -3

rise -2
run 3

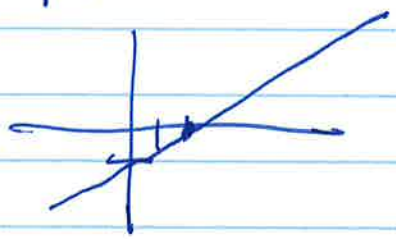


(B) Write equation of line with slope $\frac{2}{3}$ intercept -2

$$y = \frac{2}{3}x - 2$$

Matched Write eqn of line with slope $\frac{1}{2}$ y intercept -1. Graph

$y = \frac{1}{2}x - 1$



rise = 1
run = 2

Suppose a line has slope m and passes through (x_1, y_1). Let another point be x, y.

Then

$$m = \frac{y - y_1}{x - x_1}$$

Or $\boxed{y - y_1 = m(x - x_1)}$

This is point slope form of a line.

Example Find an equation for the line with slope $1/2$ passing through $(-4, 3)$

(A) Write answer in form $Ax + By = C$

$$y - 3 = \frac{1}{2}(x + 4)$$

$$2y - 6 = x + 4$$

$$-x + 2y = 10$$

(B) Find equation of line through $(-3, 2)$ and $(-4, 5)$ Write in form $y = mx + b$

$$m = \frac{2 - 5}{-3 + 4} = -3$$

$$y - 2 = -3(x + 3)$$

$$y - 2 = -3x - 9$$

$$y = -3x - 7$$

p913

matched

Find an equation for the line that has slope $\frac{2}{3}$ and passes through $(6, -2)$. Write resulting equation in form $Ax + By = C$, $A > 0$

$(y + 2) = \frac{2}{3}(x - 6)$ ↓ don't skip

$3y + 6 = 2x - 12$

$3y - 2x = -18$

$2x - 3y = 18$

Find an equation for the line that passes through (2, -5) and (4, 3)

Write in form $y = mx + b$

$$m = \frac{3+3}{4-2} = \frac{6}{2} = 3$$

$$(y+3) = m(x-2)$$

$$(y+3) = 3(x-2)$$

$$y+3 = 3x-6$$

$$\boxed{y = 3x - 9}$$

Check

$y = 3(2) - 9$

$= -3$

$y = 4(3) - 9 = 3$

Example 7: **Applications**

The management of a company that manufactures skateboards has Fixed costs of $300/day and total costs of 4,300 per day at an output of 100 skateboards per day. Assume costs are linearly related to output.

A) Find slope of equation relating cost to output

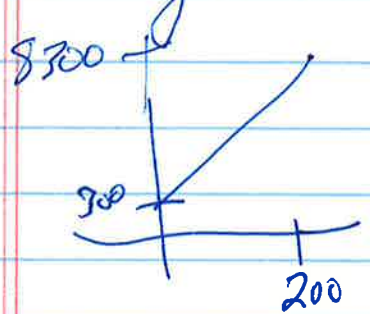
B) Find equation of line in form $C = mx + b$

C) Graph

$$m = \frac{\text{Change in cost}}{\text{Change in output}} \qquad \frac{4300 - 300}{100} = 40$$

$b = 300$ y intercept output when $x = 0$

$y = 40x + 300$



Marked 7: Answer A and B for

Fixed costs of $250/day and total costs of $450/day at 80 skateboards/day

$$m = \frac{450 - 250}{80 - 0} = \frac{200}{80} = \frac{20}{8} = 40$$

$y = 40x + 250$

In a free competitive market, the price of a product is determined by the relationship between supply and demand. If there is a surplus (supply greater than demand) price goes down. If there is a shortage (demand greater than supply) price goes up.

Price tends to move toward equilibrium price at which supply and demand are equal

Example 8: At a price of $9.00 per box of oranges the supply is 320,000 boxes and demand is 204,000 boxes, At a price of $9.50/box supply is 270,000 and demand is 300,000 boxes, Assume linear supply and demand equations

Find price-supply equation of form

$p = mx + b$ p is price, x is quantity in thousands

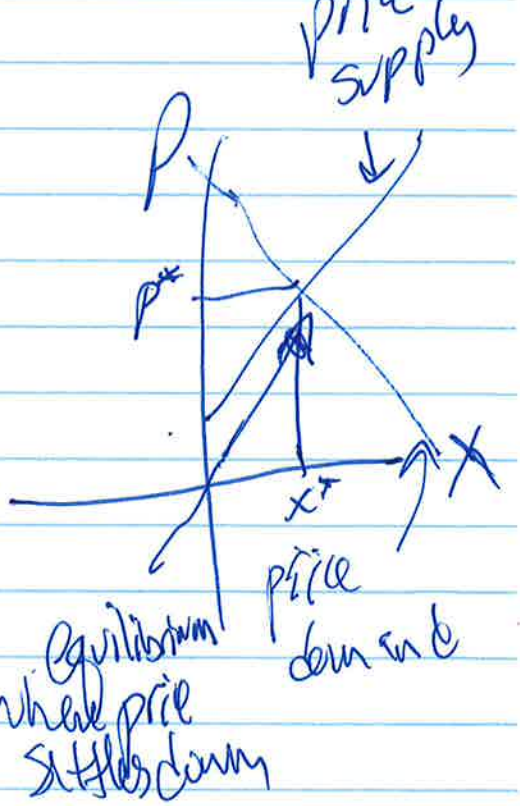
$$m = \frac{9.5 - 9}{320 - 270} = 0.01$$

$$p - p_1 = m(x - x_1)$$

$$p - 9 = 0.01(x - 320)$$

$$p - 9 = 0.01x - 3.2$$

$$p = 0.01x + 5.8$$



matched

At a price of 12.59 per box of grapefruit, the supply is 595,000 boxes and the demand is 650,000 boxes. At a price of 13.19 per box the supply is 695,000 boxes and the demand is 590,000 boxes. Assume relationships between price/supply and price/demand are linear

(A) Find price-supply equation of form $p = mx + b$

$$\frac{13.19 - 12.59}{695 - 595} = \frac{.60}{100} = 0.006$$

(x = quantity in 000's of boxes)

$$p - 12.59 = 0.006(x - 595)$$

$$p = 0.006x + (12.59 - (.006)(595))$$

$$p = 0.006x + 9.02$$

(B) Find price-demand equation of form $p = mx + b$

$$m = \frac{13.19 - 12.59}{590 - 650} = -0.01$$

9# $p - 12.59 = -0.01(x - 650)$

$p - 12.59 = -0.01x - 6.5$

$p = -0.01x + 12.59 + 6.5$

$p = -0.01x + 19.09$

Go over intercepts!