Practice Problems

1. Find and classify all critical points of the function

$$f(x) = x - \ln(x)$$

2. Find the inflection points of

$$f(x) = \frac{e^x}{1+e^x}$$

3. Demand for tickets to an amusement park is $p = 50 - .04q$. Assume costs are fixed. What price maximizes profit?

Problem 16 §4.12

4. A reasonably realistic model of a firms costs is given by the Short-run Cobb-Douglas cost curve

$$C(q) = kq^{1/a} + F$$

where a is a positive constant, F is the fixed cost, k measures the technology available to firm.

(a) Show that C is concave down if $a > 1$

(b) Assuming $a < 1$, what value of q minimizes average cost.

5. The elasticity of a good is 1.3, what is the effect of a 2% price increase?

Prob 13 section 4.6

6. The demand for yams is given by $q = 5000 - 10p^2$ where q is ~~[illegible]~~ in pounds of yams and p is the price of a pound of yams

a) If the current price of yams is $2 per pound how many pounds will be sold

(b) Is the demand at $2 elastic or inelastic

(c) What is the elasticity of demand at $2,

Solutions

1. $f'(x) = 1 - 1/x = 0$ must have $x > 0$

$1 = 1/x$

$x = 1$ critical points

$f''(x) = \frac{d}{dx}(1 - x^{-1}) = 0 + x^{-2} = \frac{1}{x^2} > 0$

Critical point at $x = 1$ is a minimum

2. $f(x) = \frac{e^x}{1+e^x}$

$$f'(x) = \frac{(1+e^x)e^x - e^x e^x}{(1+e^x)^2}$$

$$= \frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$$

$$f''(x) = \frac{(1+e^x)^2 e^x - e^x \cdot 2(1+e^x)e^x}{(1+e^x)^2} = 0$$

$$= \frac{e^x(1+e^x)\left[1+e^x - 2e^x\right]}{(1+e^x)^2}$$

inflection point where $1 - e^x = 0$

$\Rightarrow e^x = 1 \Rightarrow x = 0$

3. $p = 50 - .04q$

$50 - p = .04q$

$q = \frac{50}{.04} - \frac{1}{.04}p$

$q = 1250 - 25p$

$R = p \cdot q = 1250p - 25p^2$

Revenue maximized where profit is

$R' = 1250 - 50p$

$p = \frac{1250}{50} = 25$

$R'' = -50$ maximum

4. $C'(q) = \frac{k}{a} q^{1/a - 1}$

(a) $C''(q) = \frac{k}{a^2} q^{(1/a - 2)}$

Don't bother with (a)

(4b) $C(q) = kq^{1/a} + F$

$$a(q) = \frac{C(q)}{q} = kq^{1/a-1} + Fq^{-1}$$

$$a'(q) = k(1/a - 1)q^{(1/a-2)} + F(-1)q^{-2} = 0$$

$$\frac{F}{q^2} = k(1/a - 1)q^{(1/a-2)}$$

$$\frac{F}{k(1/a-1)} = q^{1/a-2+2}$$

$$q = \sqrt[\frac{1}{a}]{\frac{F}{k(1/a-1)}}$$

(5) $$\left|\frac{\text{Percent change in demand}}{\text{Percent change in price}}\right| = 1.3$$

$$\left|\frac{x}{2\%}\right|$$

$$x = -1.3 \cdot 2\%$$

$$= -2.6\%$$

2.6% decrease in price.

6. $q = 5000 - 10p^2$ $\quad \frac{dq}{dp} = -20p$

a) $q = 5000 - 10 \cdot 4$

$= 5000 - 40$

$= 4960$

(b) $E = \left| \frac{p}{q} \frac{dq}{dp} \right| = \left| \frac{2}{4960} \cdot -20 \cdot 2 \right|$

$= 0.0161$

inelastic

c