Homework #6

1.101 Test scores. Many states have programs for assessing the skills of students in various grades. The Indiana Statewide Testing for Educational Progress (ISTEP) is one such program.[33] In a recent year 76,531 tenth-grade Indiana students took the English/language arts exam. The mean score was 572 and the standard deviation was 51. Assuming that these scores are approximately Normally distributed, $N(572, 51)$, use the 68–95–99.7 rule to give a range of scores that includes 95% of these students.

1.102 Use the 68-95-99.7 rule. Refer to the previous exercise. Use the 68–95–99.7 rule to give a range of scores that includes 99.7% of these students.

1.103 Find the z-score. Consider the ISTEP scores (see Exercise 1.101), which we can assume are approximately Normal, $N(572, 51)$. Give the z-score for a student who received a score of 620.

1.104 Find another z-score. Consider the ISTEP scores which we can assume are approximately Normal, $N(572, 51)$. Give the z-score for a student who received a score of 510. Explain why your answer is negative even though all of the test scores are positive.

1.105 Find the proportion. Consider the ISTEP scores, which are approximately Normal, $N(572, 51)$. Find the proportion of students who have scores less than 620. Find the proportion of students who have scores greater than or equal to 620. Sketch the relationship between these two calculations using pictures of Normal curves similar to the ones given in Example 1.36.

1.106 Find another proportion. Consider the ISTEP scores, which are approximately Normal, $N(572, 51)$. Find the proportion of students who have scores between 620 and 660. Use pictures of Normal curves similar to the ones given in Example 1.37 to illustrate your calculations.

1.107 What score is needed to be in the top 25%? Consider the ISTEP scores, which are approximately Normal, $N(572, 51)$. How high a score is needed to be in the top 25% of students who take this exam?

1.108 Find the score that 80% of students will exceed. Consider the ISTEP scores, which are approximately Normal, $N(572, 51)$. Eighty percent of the students will score above x on this exam. Find x.

Solutions

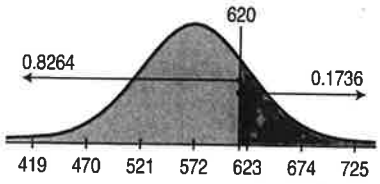
1.101. Take the mean plus or minus two standard deviations: $572 \pm 2(51) = 470$ to 674.

1.102. Take the mean plus or minus three standard deviations: $572 \pm 3(51) = 419$ to 725.

1.103. The z-score is $z = \frac{620-572}{51} \doteq 0.94$.

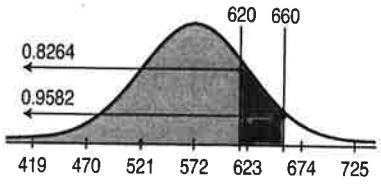
1.104. The z-score is $z = \frac{510-572}{51} \doteq -1.22$. This is negative because an ISTEP score of 510 is below average; specifically, it is 1.22 standard deviations below the mean.

1.105. Using Table A, the proportion below 620 ($z \doteq 0.94$) is 0.8264 and the proportion at or above is 0.1736; these two proportions add to 1. The graph on the right illustrates this with a single curve; it conveys essentially the same idea as the "graphical subtraction" picture shown in Example 1.36.



1.106. Using Table A, the proportion below 620 ($z \doteq 0.94$) is 0.8264, and the proportion below 660 ($z \doteq 1.73$) is 0.9582. Therefore:

area between 620 and 660	=	area left of 660	−	area left of 620
0.1318	=	0.9582	−	0.8264



The graph on the right illustrates this with a single curve; it conveys essentially the same idea as the "graphical subtraction" picture shown in Example 1.37.

1.107. Using Table A, this ISTEP score should correspond to a standard score of $z \doteq 0.67$ (software gives 0.6745), so the ISTEP score (unstandardized) is $572 + 0.67(51) \doteq 606.2$ (software: 606.4).

1.108. Using Table A, x should correspond to a standard score of $z \doteq -0.84$ (software gives -0.8416), so the ISTEP score (unstandardized) is $x = 572 - 0.84(51) \doteq 529.2$ (software: 529.1).