6.10 Margin of error and the confidence interval. A study based on a sample of size 36 reported a mean of 87 with a margin of error of 10 for 95% confidence.

(a) Give the 95% confidence interval.

(b) If you wanted 99% confidence for the same study, would your margin of error be greater than, equal to, or less than 10? Explain your answer.

6.11 Changing the sample size. Suppose that the sample mean is 50 and the standard deviation is assumed to be 7. Make a diagram similar to Figure 6.5 (page 350) that illustrates the effect of sample size on the width of a 95% interval. Use the following sample sizes: 10, 20, 40, and 80. Summarize what the diagram shows.

6.12 Changing the confidence level. A study with 49 observations had a mean of 70. Assume that the standard deviation is 14. Make a diagram similar to Figure 6.6 (page 352) that illustrates the effect of the confidence level on the width of the interval. Use 80%, 90%, 95%, and 99%. Summarize what the diagram shows.

6.13 Confidence interval mistakes and misunderstandings. Suppose 400 randomly selected alumni of the University of Okoboji were asked to rate the university's counseling services on a 1 to 10 scale. The sample mean ($\bar{x}$) was found to be 8.6. Assume that the population standard deviation is known to be $\sigma = 2.0$.

(a) Ima Bitlost computes the 95% confidence interval for the average satisfaction score as $8.6 \pm 1.96(2.0)$. What is her mistake?

(b) After correcting her mistake in part (a), she states "I am 95% confident that the sample mean falls between 8.404 and 8.796." What is wrong with this statement?

(c) She quickly realizes her mistake in part (b) and instead states "The probability the true mean is between 8.404 and 8.796 is 0.95." What misinterpretation is she making now?

(d) Finally, in her defense for using the Normal distribution to determine the confidence coefficient she says "Because the sample size is quite large, the population of alumni ratings will be approximately Normal." Explain to Ima her misunderstanding and correct this statement.

6.14 More confidence interval mistakes and misunderstandings. Suppose 100 randomly selected members of MySpace Karaoke[6] were asked how much time they typically spend on the site during the week. The sample mean ($\bar{x}$) was found to be 4.2 hours. Assume that the population standard deviation is known to be $\sigma = 2.5$.

(a) Cary Oakey computes the 95% confidence interval for the average time on the site as $4.2 \pm 1.96(2.5/100)$. What is his mistake?

(b) He corrects this mistake and then states "95% of the members spend between 3.71 and 4.69 hours a week on the site." What is wrong with his interpretation of this interval?

(c) The margin of error is slightly less than a half hour. To reduce this down to 15 minutes, Gary says the sample size needs to be doubled to 200. What is wrong with this statement?

6.10. (a) The 95% confidence interval is $87 \pm 10 = 77$ to 97. (The sample size is not needed.)
(b) Greater than 10: A wider margin of error is needed in order to be more confident that the interval includes the true mean.
Note: *If this result is based on a Normal distribution, the margin of error for 99% confidence would be roughly 13.1, because we multiply by 2.576 rather than 1.96.*

6.11. The margins of error are $1.96 \times 7/\sqrt{n}$, which yields 4.3386, 3.0679, 2.1693, and 1.5339. (And, of course, all intervals are centered at 50.) Interval width decreases with increasing sample size.
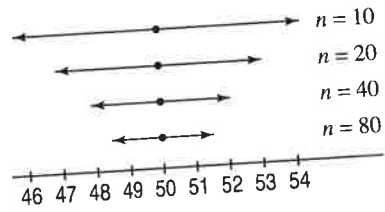
6.12. The margins of error are $z^* \times 14/\sqrt{49} = 2z^*$. With z^* equal to 1.282, 1.645, 1.960, and 2.576, this yields 2.564, 3.290, 3.920, and 5.152. (And, of course, all intervals are centered at 70.) Increasing confidence makes the interval wider.
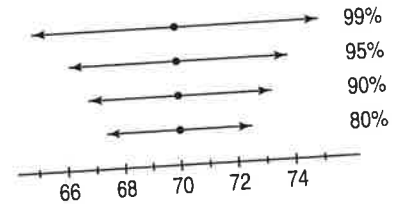
6.13. (a) She did not divide the standard deviation by $\sqrt{n} = 20$. **(b)** Confidence intervals concern the population mean, not the sample mean. (The value of the sample mean is known to be 8.6; it is the population mean that we do not know.) **(c)** 95% is a confidence level, not a probability. Furthermore, it does not make sense to make probability statements about the population mean μ, which is an unknown constant (rather than a random quantity). **(d)** The large sample size does not affect the distribution of individual alumni ratings (the population distribution). The use of a Normal distribution is justified because the distribution *of the sample mean* is approximately Normal when the sample is large.
Note: *For part (c), a Bayesian statistician might view the population mean μ as a random quantity, but the viewpoint taken in the text is non-Bayesian.*

6.14. (a) The standard deviation should be divided by $\sqrt{100} = 10$, not by 100. **(b)** The correct interpretation is that (with 95% confidence) the *average* time spent at the site is between 3.71 and 4.69 hours. That is, the confidence interval is a statement about the population mean, not about the individual members. **(c)** To halve the margin of error, the sample size needs to be quadrupled, to about 400. (In fact, $n = 385$ would be enough.)