Math 211
Spring 2015
Exam 1 Practice 2
2/13/15
Time Limit: 75 Minutes

Name (Print): Solutions

This exam contains 9 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. Graphing calculators should not be needed, but they can be used to check your work. If you use a graphing calculator to find an answer you must write the steps needed to find the answer, without the calculator.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	5	
4	5	
5	5	
6	5	
7	20	
8	20	
Total:	100	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1} \\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x \\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g+fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x) \\
f(x) &\approx f(a)+f'(a)(x-a)
\end{aligned}$$

1. (20 points) Consider the function $f(x) = 3x + e^x$.

   (a) (5 points) What is average rate of change of f between $x = 1$ and $x = 3$?

$$\frac{F(3)-F(1)}{3-1} = \frac{9+e^3-3-e^1}{2}$$

$$= \frac{6+e^3-e}{2}$$

   (b) (5 points) What is the instantaneous rate of change of f at $x = 1$?

$$F'(x) = 3 + e^x$$

$$F'(1) = 3 + e$$

   (c) (5 points) What is the relative rate of change of f at $x = 1$?

$$\frac{F'(1)}{F(1)} = \frac{3+e}{3+e} = 1$$

   (d) (5 points) Is f concave up, concave down, or neither, at $x = 1$?

$$F''(x) = e^x$$

$$F''(1) = e$$

Concave up

2. (20 points) The demand curve for a product is given by $q = 40 - p$. The supply curve is given by $q = 8 + 3p$.

   (a) (5 points) Find the equilibrium price.

$$40 - p^* = 8 + 3p^*$$
$$32 = 4p^*$$
$$p^* = 8$$

   (b) (5 points) Find the equilibrium quantity.

$$q^* = 40 - 8 = 32$$

Check

$$q^* = 8 + 3p^* = 32$$

(c) (5 points) Find the new equilibrium price if a 4% tax is charged to the consumer.

Consumer pays more

supply $q = 8 + 3p$ (same)

demand $q = 40 - (1.04)p$

$8 + 3p = 40 - 1.04p$

$4.04p = 32$

$p = \frac{32}{4.04} = 7.92$ price before tax

$7.92 + 4\%$ price after tax

$= 8.24$

(d) (5 points) Find the new equilibrium quantity if a 4% tax is charged to the consumer.

$q = 8 + 3p = 8 + 3(7.92)$

$= 31.76$

3. (5 points) Find the future value in 10 years of \$10,000 invested today at an interest rate of 5%, under continuous compounding.

$10000e^{0.05 \cdot 10}$

$= 16487.21$

4. (5 points) Find the present value of a \$10,000 payment, expected in 10 years, assuming an interest rate of 5%, under continuous compounding.

$10000e^{-0.05 \cdot 10}$

$= 6065.30$

5. (5 points) Find the equation of the tangent line to the function $f(x) = 3x + e^x$ at $x = 5$.

$F(5) = 3 \cdot 5 + e^5 = 15 + e^5$

$F'(x) = 3 + e^x$

$F'(5) = 3 + e^5$

Line through $(5, 15 + e^5)$ slope $3 + e^5$

$y = 15 + e^5 + (3 + e^5)(x - 5)$

6. (5 points) The total cost of producing q units of a good is $C(q) = 500 + 3q^3 + e^q$. What are the fixed costs? What is the marginal cost of producing the 10^{th} unit of the good?

Fixed costs $C(0) = 500 + 0 + e^0 = 501$

Marginal Costs $C'(q) = 9q^2 + e^q$

$C'(10) = 9 \cdot 100 + e^{10}$

$= 900 + e^{10}$

7. (20 points) The marginal cost of producing neato widgets is \$8 per widget (independent of quantity). The fixed cost is \$150. The widgets sell for \$20 each.

   (a) (5 points) Find the cost function relating total cost to number of widgets produced.

$C(q) = 150 + 8q$

   (b) (5 points) Find the revenue function relating revenue to number of widgets sold.

$R(q) = 20q$

   (c) (5 points) Find the profit function relating profit to number of widgets produced, assuming everything produced sells.

$\Pi(q) = R(q) - C(q) = 20q - 150 - 8q$

$= 12q - 150$

   (d) (5 points) Find the break even point.

$\Pi(q_{be}) = 0$

$12q - 150 = 0$

$q = \frac{150}{12} = 12.5$

8. (20 points) Take the derivatives of the following:

(a) (4 points) $y = \sin(\cos(x))$

$$\frac{dy}{dx} = \cos(\cos x)(-\sin(x))$$

(b) (4 points) $y = xe^{\cos(x)}$

$$\frac{dy}{dx} = e^{\cos(x)} + xe^{\cos(x)}(-\sin(x))$$

(c) (4 points) $y = \frac{\sin(x^2)}{\cos(x^2)}$

$$\frac{dy}{dx} = \frac{\cos(x^2)\cos(x^2)(2x) - \sin(x^2)(-\sin(x^2))(2x)}{(\cos(x^2))^2}$$

(d) (4 points) $y = \ln(x)\cos(e^x)$

$$\frac{dy}{dx} = \frac{1}{x}\cos(e^x) + \ln x \cdot (-\sin(e^x))e^x$$

(e) (4 points) $y = \cos(\sin(\ln(x)))$

$$\frac{dy}{dx} = -\sin(\sin(\ln(x)))\cos(\ln(x)) \cdot \frac{1}{x}$$