Math 211-20155-W8-Tuesday

§5.1 Distance and Accumulated Change

Question: IF you drive 3 hours at 50 mph how far have you gone

$$50 \text{ mph} \cdot 3 \text{ hr} = 150 \text{ mi}$$

$$\frac{\text{miles}}{\cancel{\text{hour}}} \cdot \cancel{\text{hour}}$$

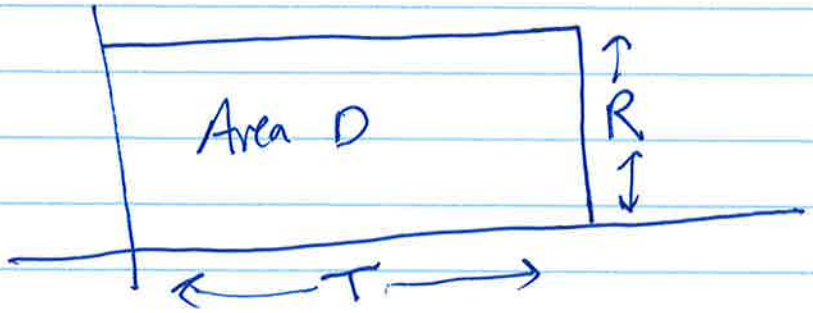
IF you drive for t hours at r mph how far have ya gone

$$t \cdot r$$

Distance traveled = Velocity × Time

$$D = R \cdot T$$

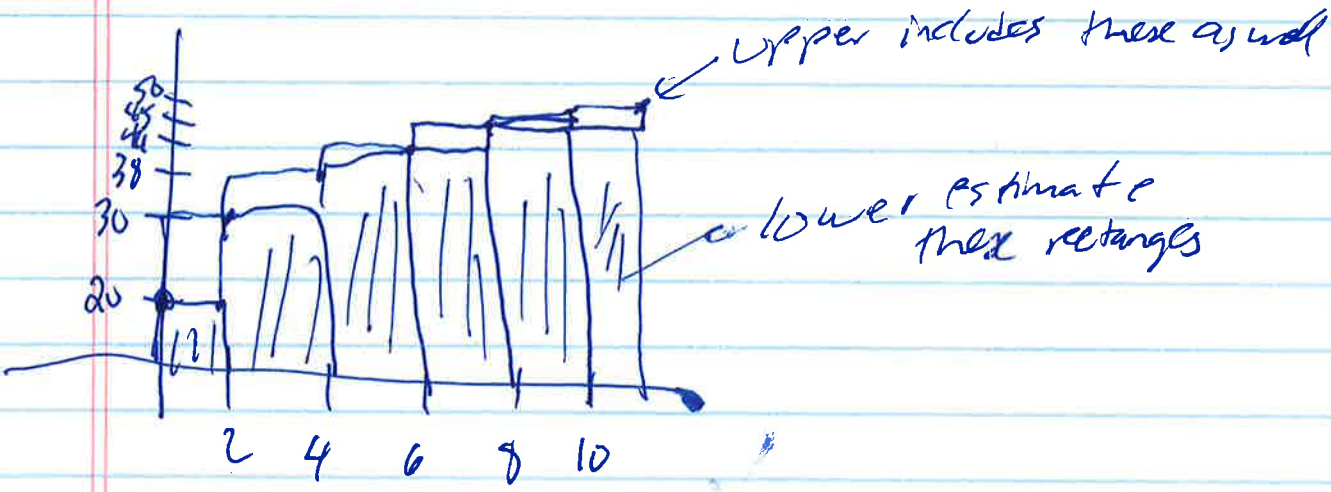
We can display this graphically as follows



Distance is area under velocity graph

Say car is moving with increasing velocity (velocity not constant

Time (sec)	0	2	4	6	8	10
Vel (ft/sec)	20	30	38	44	~~48~~	50



In interval between time = 0
and t = 20
Velocity ~~Speed~~ is between 0 and 30
this distance traveled is between 40 and 60

$D = R \cdot T$
R: between 20 and 30
T: 2

We could do this for all intervals and get a lower estimate
$= 20 \cdot 2 + 30 \cdot 2 + 38 \cdot 2 + 44 \cdot 2 + 48 \cdot 2 = 360$ ft
and an ~~B~~ upper estimate
$= 30 \cdot 2 + 38 \cdot 2 + 44 \cdot 2 + 48 \cdot 2 + 50 \cdot 2 = 420$ ft

What if you had velocity data every second?

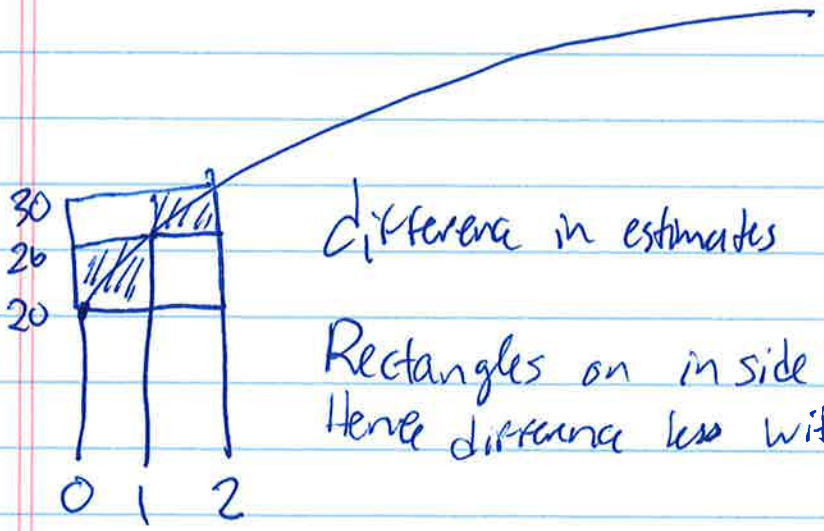
0 1. 2. 3. 4. 5. 6. 7. 8. 9. 10
20 26. 30. 35. 38. 42. 44. 46. 48. 49. 50

New lower estimate 378 ft
New upper estimate 408 ft

Note

$$360 < 378 < 408 < 420$$

More frequent data means a better estimate.



difference in estimates is area of rectangle

Rectangles on inside have less total area. Hence difference less with more data

If you continue to take smaller and smaller rectangles the upper estimate and lower estimate will come together

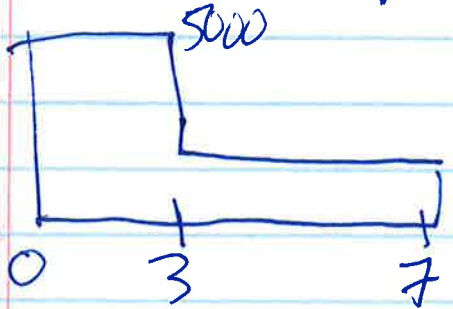
OR in the language of calculus
"converge to a limit"

The limit is the distance traveled
aka "the integral of velocity"

If a function is the rate of change
(eg velocity)
its integral is the total change
(eg distance)

Eg: A city's population grows at a rate of 5000 people/year for 3 years then grows at a rate of 3000 people/year for the next 4 years,
What is the total change in population

Total change = Rate of change × Time



5000 · 3
+ 3000 · 4
= 27000 more people
(total change)