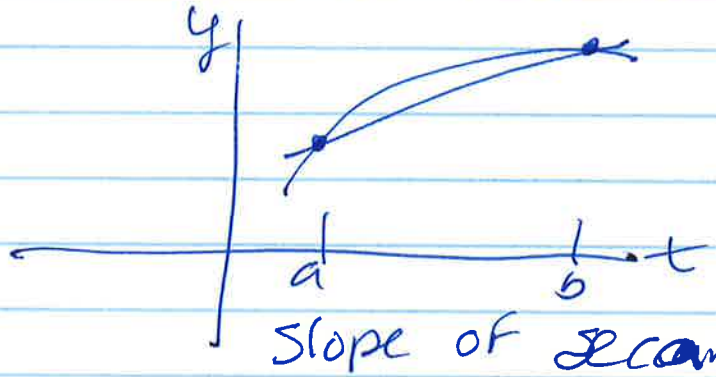
~~Applications of Derivatives~~

Review

Average rate of change of a function between a and b

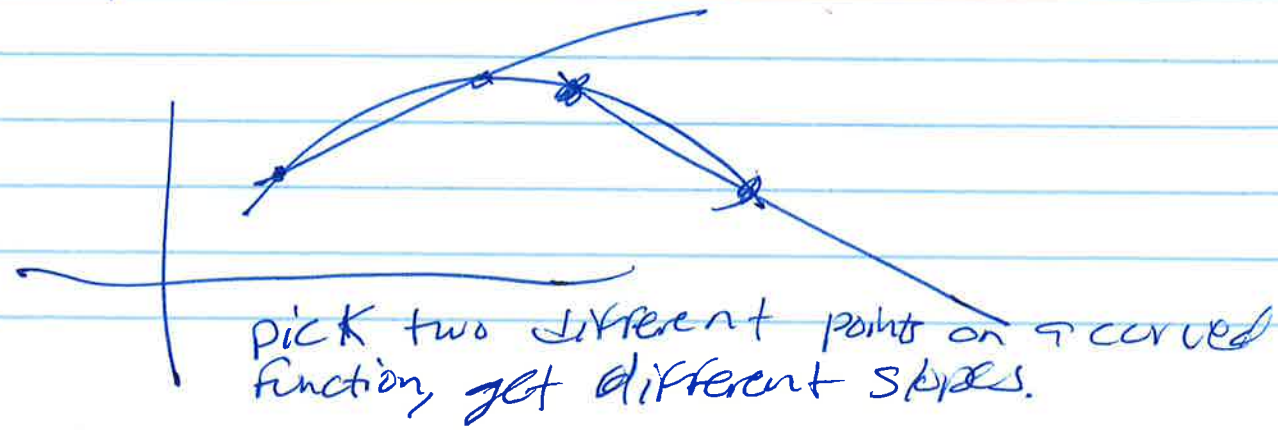


slope of secant line

What if a line went through those two points? What would its slope be?

$$\frac{\Delta y}{\Delta t} = \frac{f(b) - f(a)}{b - a}$$ = average rate of change of f between a and b.

t often used as independent variable



pick two different points on a curved function, get different slopes.

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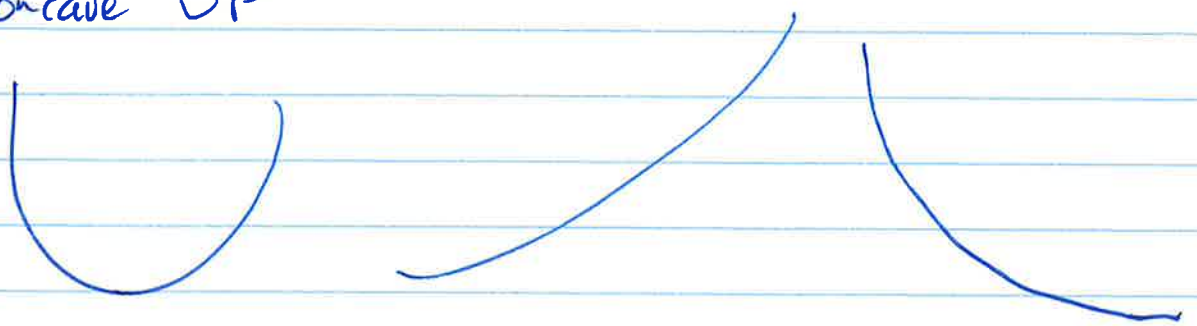
Concavity

Concave UP



Concave Down

Relative change

If P changes from P_0 to P_1

The Relative Change in $P = \frac{\Delta P}{P_0}$

$= \frac{P_1 - P_0}{P_0}$

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New: Applications of Functions to Economics.

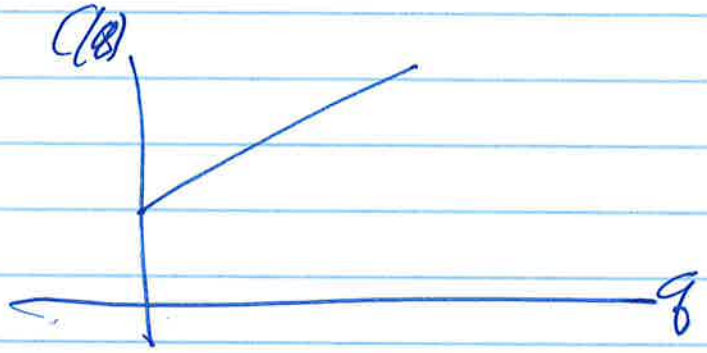
The cost Function

$C(q)$ gives the total cost of producing a quantity q of some good

$C(q)$ (the cost function) can be linear or nonlinear (curved)

Later in the semester we are going to consider curved/nonlinear cost functions

For now they look like



1) Why does the graph start at $q=0$
2) What does the vertical intercept represent?

Called Fixed Costs

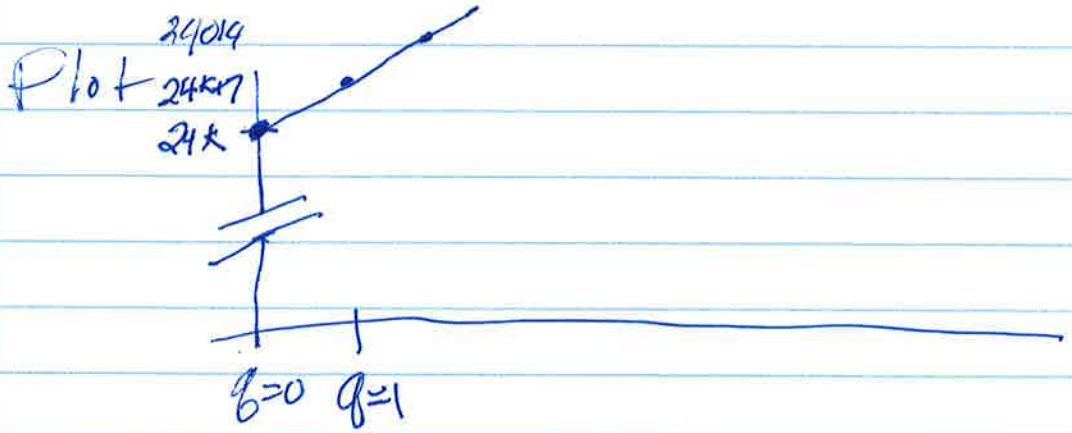
The Fixed cost is the capital investment needed before any good is produced

Example from book:

We want to start a business that makes radios.

It costs $24,000 for the machinery and equipment needed to produce the radios (even if we don't make any radios).

After we have the equipment, it cost $7 dollars to make each radio



The Fixed cost is the vertical intercept
The slope is

$$\frac{\text{Rise}}{\text{Run}} = \frac{\$7}{1 \text{ unit}} = 7$$

The slope is called the **marginal cost.**

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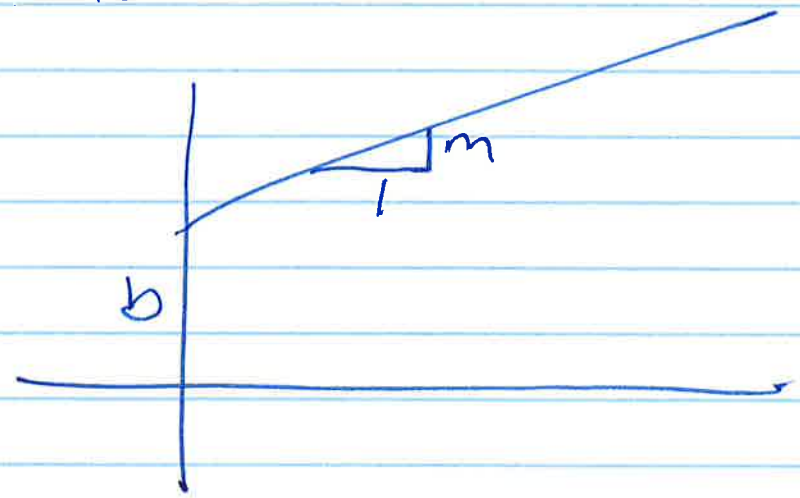
We have marginal cost and fixed cost.

slope intercept

For a linear cost function that's all there is



$y = mx + b$ familiar letters

$C = mq + b$ letters used typically with cost function (book uses numbers for m and b)

C ↑ cost, q ↑ quantity

$C = 24000 + 7q$

we haven't defined slope of nonlinear function but mention but don't emphasize

When linear, the marginal cost is the cost to produce one more unit of the good or service, (this is approx true when nonlinear) ~~When nonlinear the MC is still the slope of the cost funct, but this slope changes.~~

Can you think of good or service where the Fixed cost is high but the marginal cost is low or zero?

Can you think of a good or service where the fixed cost is low or zero but the marginal cost is high?

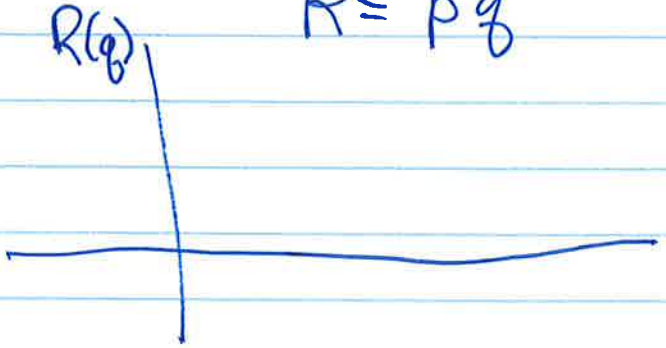
Costs are not the whole story

The Revenue Function $R(q)$ gives the total revenue received by a firm from selling a quantity q of some good.

If the good sells for price p

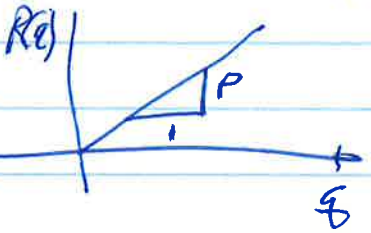
Revenue = Price × Quantity

$R = pq$



1) Is the $R(q)$ a linear function of (q) for fixed price
2) What's vertical intercept?
3) What's slope

The price is sometimes called the marginal Revenue



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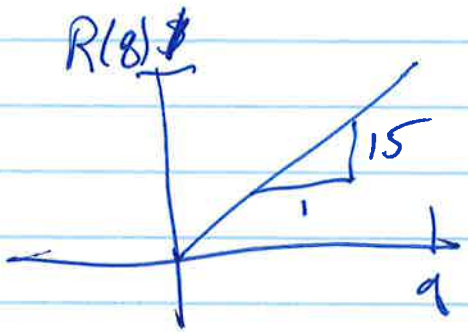
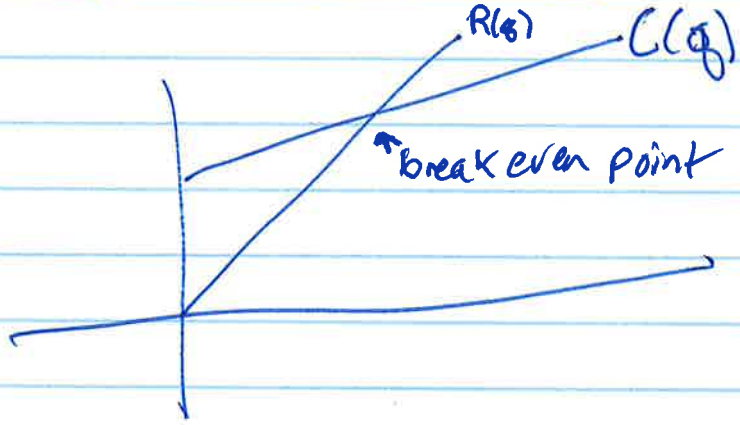
IF radios sell for $15/each



Breakeven point

Cost = Revenue



$C(q) = R(q)$

Radio Example $2000 + 7q = 15q$

$2000 = 8q$

$q = 3000$ → must sell 3000 radios to breakeven

if sell less, lose money

Profit Function

Profit is the amount of money earn after subtracting costs,

$$\text{Profit} = \text{Revenue} - \text{Costs}$$

In symbols

$$\Pi = R - C$$

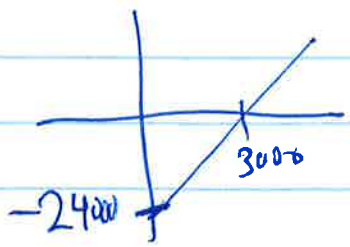
$$\Pi(q) = R(q) - C(q)$$

The greek letter Π (Pi) is used because "p" is already used for "price".

Profit for radios

$$\Pi(q) = 15q - (24000 + 7q)$$

$$= 8q - 24000$$



break even where $\Pi(q) = 0$

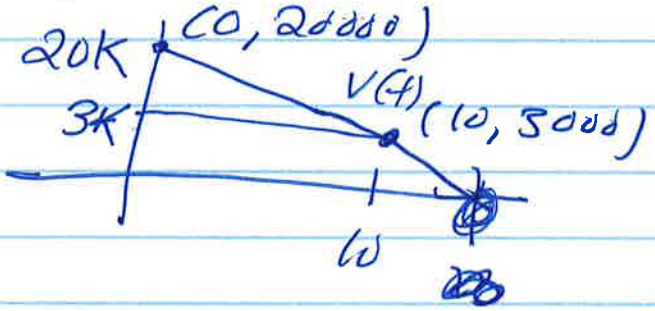
The depreciation function

Machinery tends to lose its value over time.

IF a machine cost $20000 today and sells for $3000 ten years from now we say the machine depreciates from $20K to $3K.

We can write a depreciation function

Assume it is linear



Find a formula for V(t)

$$V(t) = mt + b$$

$$m = \frac{3000 - 20000\ \$}{10\ \text{year}} = 1700\ \$/\text{year}$$

$$b = 20K$$

$$V(t) = 20{,}000 - 1700t$$

dollars, t in years