

§1.1

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Math 151-2015XB Day 1 W1

Ch1 - Linear equation and graph

§1.1 Linear Equation and Inequalities

Linear - we won't worry about that now

Equation
Inequality } What are these things?

Equation - a formula that equates two things $3x + 2 = 5$
(with an equals sign)

Inequality - a formula that relates two (sometimes three) things by one of $<, \leq, \neq, >, \geq$ these signs.

Examples $3x + 2 \geq 5$
 $-2 \leq x < 2$

What is this x ?

Equations and Inequalities usually have one or more ~~unknowns~~ (variables) (often just x) Sometimes different letters are used, sometimes two or more ~~variables~~ ~~unknowns~~ are present in the same equation.

An equation in one variable has ~~one~~ ~~unknown~~ (e.g. x) (one variable)

(inequality)

A solution of an equation involving a single variable is a number that when substituted for the variable ~~(unknown)~~ make the equation (inequality) true

$$2x = 8$$

$$\text{Solution : } x = 4$$

$$x^2 = 4$$

$$\text{Solutions } x = 2 \text{ and } x = -2$$

The set of solutions is the solution set.

Two equations or inequalities are said to be equivalent if they have the same solution sets.

"Theorem 1" in book

The main result for doing algebra:

An equivalent equation (not inequality) will result if,

- * The same quantity is added to or subtracted from each side of a given equation
- * Each side of a given equation is multiplied by or divided by the same non zero quantity.

Solving a linear equation

Solve and Check

$$8x - 3(x - 4) = 3(x - 4) + 6$$

Distrib

$$8x - 3x + 12 = 3x - 12 + 6$$

Combine

$$5x + 12 = 3x - 6$$

-3x both sides

$$2x + 12 = -6$$

-12 both sides

$$2x = -18$$

÷2 both sides

$$x = -9$$

Check:

$$8(-9) - 3(-9 - 4) \stackrel{?}{=} 3(-9 - 4) + 6$$

$$-72 - 3(-13) \stackrel{?}{=} 3(-13) + 6$$

$$-72 + 39 \stackrel{?}{=} -39 + 6$$

$$33 \checkmark 33$$

Matched

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$$3x - 2(2x - 5) = 2(x + 3) - 8$$

Distrib

$$3x - 4x + 10 = 2x + 6 - 8$$

Combine

$$-x + 10 = 2x - 2$$

+x both sides

$$10 = 3x - 2$$

+2 both sides

$$12 = 3x$$

÷3 both sides

$$4 = x$$

Check:

$$\begin{aligned} 3(4) - 2(2(4) - 5) &\stackrel{?}{=} 2(4 + 3) - 8 \\ 12 - 2(8 - 5) &\stackrel{?}{=} 2(7) - 8 \\ 12 - 2(3) &\stackrel{?}{=} 14 - 8 \\ 12 - 6 &\stackrel{?}{=} 6 \\ 6 &\stackrel{?}{=} 6 \end{aligned}$$

Example 2: Solve and check

$$\frac{x+2}{2} - \frac{x}{3} = 5$$

How do we eliminate denominators
both sides

* Multiply [^] by any common denominator
The simplest is the least common denominator, in this case 6;
Multiply both sides by 6

$$\left(\frac{x+2}{2} - \frac{x}{3} \right) 6 = 5 \cdot 6$$

Simplify $(x+2) \frac{6}{2} - x \cdot \frac{6}{3} = 30$

$$(x+2) 3 - x \cdot (2) = 30$$

Distrib

$$3x+6 - 2x = 30$$

Combine

$$x+6 = 30$$

subtract 6
both sides

$$x = 24$$

check $\frac{24+2}{2} - \frac{24}{3} \stackrel{?}{=} 5$

$$\frac{26}{2} - 8 \stackrel{?}{=} 5$$

$$13 - 8 \stackrel{?}{=} 5$$

$$5 = 5$$

Matched

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Solve and check

$$\frac{x+1}{3} - \frac{x}{4} = \frac{1}{2}$$

$$\times 12$$

$$(x+1)\frac{12}{3} - x\left(\frac{12}{4}\right) = \frac{12}{2}$$

Smp

$$(x+1)4 - x \cdot 3 = 6$$

$$4x + 4 - 3x = 6$$

$$x + 4 = 6$$

$$x = 2$$

check

$$\frac{2+1}{3} - \frac{2}{4} \stackrel{?}{=} \frac{1}{2}$$

$$\frac{3}{3} - \frac{1}{2} \stackrel{?}{=} \frac{1}{2}$$

$$1 - \frac{1}{2} \stackrel{?}{=} \frac{1}{2}$$

$$\frac{1}{2} \stackrel{\checkmark}{=} \frac{1}{2}$$

Example 3: If you deposit a principle P in an account that earns simple interest at an annual rate r then the amount A in the account after t years is given by

$$A = P + Prt \quad 4 \text{ variables}$$

(A) write r in terms of A, P, t

$$A = P + Prt$$

$$P + Prt = A$$

$$Prt = A - P$$

$$r = \frac{A - P}{Pt}$$

reverse
subt P
 $\div rt$

(B) write P in terms of A, r, t

$$P + Prt = A$$

$$P(1 + rt) = A$$

$$P = \frac{A}{1 + rt}$$

factor out P
 $\div 1 + rt$

matched

If a cardboard box has length L , width W and height H then its surface area is given by the formula

$$S = 2LW + 2LH + 2WH$$

(A) solve for L in terms of S, W, H

factor
 $2L$

$$S = 2L(W+H) + 2WH$$

Reverse

$$2L(W+H) + 2WH = S$$

sub $2WH$

$$2L(W+H) = S - 2WH$$

$\div 2(W+H)$

$$L = \frac{S - 2WH}{2(W+H)}$$

A Linear equation is one that can be written in the following standard form

$$ax + b = 0 \quad a \neq 0$$

Linear inequalities

A linear inequality results when you replace $=$ with $<$, $>$, $\geq$, or $\leq$
(a is less than b)

$a < b$ means there exists ~~there~~ a positive number p such that

$$a + p = b$$

$b > a$ means $a < b$

Example 4

(A) $3 ? 5$ $<$ because $3 + 2 = 5$

(B) $-6 ? -2$ $<$ because $-6 + 4 = -2$

(C) $0 ? -10$ $>$ because $-10 + 10 = 0$

Matched

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(A) $2 ? 8$

(B) $-20 ? 0$

(C) $-3 ? -30$

$-30 < -3$

multiply by a positive number

eg (2) $-60 < -6$ true

multiply by a negative number

eg (1) $30 ? 3$

$30 < 3$ NO

$30 > 3$

Multiply by a negative number reverse sign

"Theorem 2" - Inequalities

An equivalent inequality (same solution set) will result with the same sense or direction if each side of the original inequality

- has the same number added or subtracted from it
- is multiplied or divided by the same positive number

An equivalent inequality will result with the sense or direction reversed if each side of the original inequality

- is multiplied or divided by the same negative number

We can perform essentially the same operations on inequalities that we perform on equations, with the exception that the sense of the inequality must reverse if we multiply ~~by~~ or divide both sides by a negative number, otherwise the sense will not change.

double inequality

$a < x < b$ means $a < x$ and $x < b$
(and necessarily $a < b$)

Inequalities can also be described by interval notation and like graphs

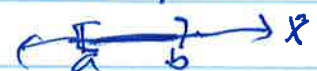
Interval
Notation

Inequality
Notation

Line
Graph

$[a, b]$

$a \leq x \leq b$



$[a, b)$

$a \leq x < b$



$(a, b]$

$a < x \leq b$



(a, b)

$a < x < b$



$(-\infty, a]$

$x \leq a$



$(-\infty, a)$

$x < a$



$[b, \infty)$

$b \leq x$



(b, ∞)

$b < x$



matched

(A) write $[-7, 4]$ as a double inequality and graph

$$-7 \leq x \leq 4$$


(B) write $x < 3$
in interval notation
and graph



$$(-\infty, 3)$$

Example 6

Solving a linear inequality

$$2(2x+3) < 6(x-2) + 10$$

Distrib

$$4x + 6 < 6x - 12 + 10$$

Combine

$$4x + 6 < 6x - 2$$

Sub $6x$

$$-2x + 6 < -2$$

Sub 6

$$-2x < -8$$

Divide -2

$$x > 4$$

must reverse sense,

$$\begin{array}{c} \text{---} \longrightarrow \\ 4 \end{array} \quad (4, \infty)$$

Matched

Solve and graph

$$3(x-1) \leq 5(x+2) - 5$$

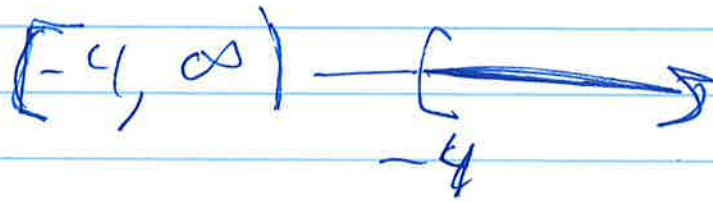
$$3x - 3 \leq 5x + 10 - 5$$

$$3x - 3 \leq 5x + 5$$

$$-2x - 3 \leq 5$$

$$-2x \leq 8$$

$$x \geq -4$$



Example 7; Solving a double inequality

$$-3 < 2x + 3 \leq 9$$

Sub 3 $-3 - 3 < 2x \leq 9 - 3$

Combine $-6 < 2x \leq 6$

Div 2 $-3 < x \leq 3$



Matched

$$-8 \leq 3x - 5 < 7$$

Add 5 $-8 + 5 \leq 3x - 5 + 5 < 7 + 5$

Combine $-3 \leq 3x < 12$

Div 3 $-1 \leq x < 4$



~~$[-1, 4)$~~ $[-1, 4)$

Applications

- Look for unknown quantity ~~#~~ (often called x)
- Identify other quantities
- Write a verbal statement of condition, in problem then a mathematical one
- Solve equation or inequality / answer questions posed in problem
- Check solution

Example 8: [Alex purchases a plasma TV, pays 7% tax
\$65 delivery. Total is \$1668.93
What is purchase price?

Step 1 Variable for unknown

x = purchase price

Knowns:

Delivery = \$65

Sales tax = $0.07x$

total price = 1668.93

Verbal statement

Price + Delivery Charge + Sales Tax = Total

$x + 65 + 0.07x = 1668.93$

Solve: Combine $1.07x + 65 = 1668.93$

Sub 65 $1.07x = 1603.93$

Div 1.07 $x = 1499.00$

Check answer

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Matched 8

Mary paid 8.5% sales tax
and \$190 title and licence fee
when she bought a car for 28,400 total

What is purchase price of car

Unknown: x = purchase price

Knowns: 0.085X tax

~~licence fee~~ 190 licence fee

~~total~~ 28,400 total

~~total~~ purchase price + tax + licence fee = total

~~28,400~~ $x + 0.085x + 190 = 28400$

Combine $1.085x + 190 = 28400$

Div 1.085 $1.085x = 28210$

$x = 26,000$

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A manufacturing company has costs

Fixed costs (plant overhead, product design, setup, promotion) does not depend on quantity produced

Variable costs (raw materials, etc) ~~do~~ do depend on the quantity produced

Example: A multimedia company produces DVDs. One-time fixed costs for a particular DVD are \$48,000 which include cost such as filming, editing and promotion. Variable cost amount to \$12.40 per DVD and include manufacturing, packaging and distribution

Fixed costs 48,000

Variable costs 12.40 X

where X is the number produced

$$\text{Costs} = 48,000 + 12.40 X$$

Lets say they sell each DVD for \$17.40

$$\text{Revenue} = 17.40 X$$

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produce

IF they don't sell any DVD
they lose 48,000

IF they sell a lot they make money

IF $R > C$ make money
 $R < C$ lose money
 $R = C$ break even

How many do they have to sell
to break even?

$$\begin{array}{l} 17.40x = 48000 + 12.40x \\ \text{Sub } 12.40x \quad 5x = 48000 \end{array}$$

$$x = 9,600$$

must sell 9600 to break even,

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Matched 9 How many DVDs would a multimedia company have to make and sell to break even if fixed costs are \$36,000 variable costs are \$10.40 per DVD and DVDs sell to retailers for \$15.20 each

Unknown x quantity produced/sold to break even.
Knowns 36,000 fixed cost
10.40 x variable cost
15.20 x Revenue

~~Cost~~ Revenue = Cost

$$\begin{array}{lcl} 15.20x & = & 36000 + 10.40x \\ \text{Sub } 10.40 & & 4.8x = 36000 \\ \text{div } 4.8 & & x = 7500 \end{array}$$

Example 10 Consumer price Index

The Consumer Price index is a measure of the average change in prices over time from a designated ~~for~~ reference period, which equals 100

CPI in 1960 29.6
CPI in 2012 229.6

What net annual salary in 2012 would have the same purchasing power as a net annual salary of 13,000 in 1960?

Unknown Let x be purchasing power in 2012
Known Annual salary in 1960 = 13000
and CPI

Key insight is ratios are the same

$$\frac{x}{13000} = \frac{229.6}{29.6}$$

mult
13000

$$x = \frac{229.6}{29.6} (13000) = 100,838 \text{ year}$$

check ratios $\frac{100,838}{13000} = 7.757 = \frac{229.6}{29.6}$

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matched 1:1

What annual salary in 1973 would have the same purchasing power as a net annual salary of 100,000 in 2012?

Unknowns: X = Annual salary 1973

Knowns Salary in 2012 = 100,000

CPI 2012 = 229.6

CPI 1973 = 44.4

Ratios
equal

$\left\{ \begin{array}{l} \boxed{1973} \\ \boxed{2012} \end{array} \right\}$
~~1973~~

$$\frac{X}{100,000} = \frac{44.4}{229.6}$$

must
have years
consistent

1973 - numerator
2012 - denominator

$$X = 19,338$$