# Homework #7 - Stat 202

**1.122 Pregnancies and the 68–95–99.7 rule.** The length of human pregnancies from conception to birth varies according to a distribution that is approximately Normal with mean 266 days and standard deviation 16 days. Use the 68–95–99.7 rule to answer the following questions.

**(a)** Between what values do the lengths of the middle 95% of all pregnancies fall?

**(b)** How short are the shortest 2.5% of all pregnancies? How long do the longest 2.5% last?

**1.126 Find some proportions.** Using either Table A or your calculator or software, find the proportion of observations from a standard Normal distribution that satisfies each of the following statements. In each case, sketch a standard Normal curve and shade the area under the curve that is the answer to the question.

**(a)** $Z > 1.65$

**(b)** $Z < 1.65$

**(c)** $Z > -0.76$

**(d)** $-0.76 < Z < 1.65$

**1.128 Find some values of $z$.** Find the value $z$ of a standard Normal variable $Z$ that satisfies each of the following conditions. (If you use Table A, report the value of $z$ that comes closest to satisfying the condition.) In each case, sketch a standard Normal curve with your value of $z$ marked on the axis.

**(a)** 22% of the observations fall below $z$.

**(b)** 40% of the observations fall above $z$.

**1.131 High IQ scores.** The Wechsler Adult Intelligence Scale (WAIS) is the most common IQ test. The scale of scores is set separately for each age group and is approximately Normal with mean 100 and standard deviation 15. The organization MENSA, which calls itself "the high IQ society," requires a WAIS score of 130 or higher for membership. What percent of adults would qualify for membership?

*There are two major tests of readiness for college, the ACT and the SAT. ACT scores are reported on a scale from 1 to 36. The distribution of ACT scores are approximately Normal with mean $\mu = 21.5$ and standard deviation $\sigma = 5.4$. SAT scores are reported on a scale from 600 to 2400. The SAT scores are approximately Normal with mean $\mu = 1509$ and standard deviation $\sigma = 321$. Exercises 1.132 to 1.141 are based on this information.*

**1.132 Compare an SAT score with an ACT score.** Tonya scores 1820 on the SAT. Jermaine scores 29 on the ACT. Assuming that both tests measure the same thing, who has the higher score? Report the $z$-scores for both students.

* see highlight

**1.134 Find the ACT equivalent.** Jose scores 2080 on the SAT. Assuming that both tests measure the same thing, what score on the ACT is equivalent to Jose's SAT score?

**1.136 Find an SAT percentile.** Reports on a student's ACT or SAT usually give the percentile as well as the actual score. The percentile is just the cumulative proportion stated as a percent: the percent of all scores that were lower than this one. Maria scores 2090 on the SAT. What is her percentile?

**1.138 How high is the top 10%?** What SAT scores make up the top 10% of all scores?

**1.142 Do you have enough "good cholesterol?"** High-density lipoprotein (HDL) is sometimes called the "good cholesterol" because low values are associated with a higher risk of heart disease. According to the American Heart Association, people over the age of 20 years should have at least 40 mg/dL of HDL cholesterol.[36] U.S. women aged 20 and over have a mean HDL of 55 mg/dL with a standard deviation of 15.5 mg/dL. Assume that the distribution is Normal.

**(a)** What percent of women have low values of HDL (40 mg/dL or less)?

**(b)** HDL levels of 60 mg/dL are believed to protect people from heart disease. What percent of women have protective levels of HDL?

**(c)** Women with more than 40 mg/dL but less than 60 mg/dL of HDL are in the intermediate range, neither very good or very bad. What proportion are in this category?

**1.145 Length of pregnancies.** The length of human pregnancies from conception to birth varies according to a distribution that is approximately Normal with mean 266 days and standard deviation 16 days.

**(a)** What percent of pregnancies last less than 240 days (that's about 8 months)?

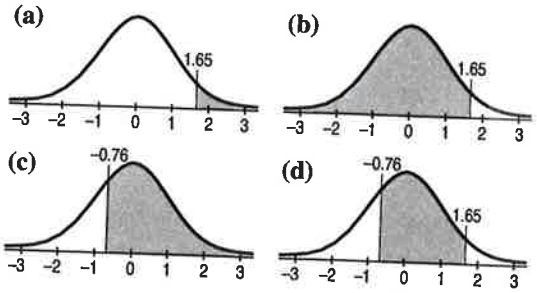
**(b)** What percent of pregnancies last between 240 and 270 days (roughly between 8 months and 9 months)?

**(c)** How long do the longest 20% of pregnancies last?

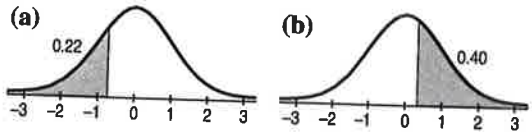
Solutions

**1.122.** See the sketch of the curve in the solution to Exercise 1.120. **(a)** The middle 95% fall within two standard deviations of the mean: $266 \pm 2(16)$, or 234 to 298 days. **(b)** The shortest 2.5% of pregnancies are shorter than 234 days (more than two standard deviations below the mean).

**1.126.** Using values from Table A:
**(a)** $Z > 1.65$: 0.0495. **(b)** $Z < 1.65$: 0.9505. **(c)** $Z > -0.76$: 0.7764. **(d)** $-0.76 < Z < 1.65$: $0.9505 - 0.2236 = 0.7269$.




**1.128.** **(a)** 22% of the observations fall below $-0.7722$. (This is the 22nd percentile of the standard Normal distribution.) **(b)** 40% of the observations fall above 0.2533 (the 60th percentile of the standard Normal distribution).




**1.131.** 130 is two standard deviations above the mean (that is, it has standard score $z = 2$), so about 2.5% of adults would score at least 130.

**1.132.** Tonya's score standardizes to $z = \frac{1820 - 1509}{321} \doteq 0.9688$, while Jermaine's score corresponds to $z = \frac{29 - 21.5}{5.4} \doteq 1.3889$. Jermaine's score is higher.

**1.134.** Jose's score standardizes to $z = \frac{2080 - 1509}{321} \doteq 1.7788$, so an equivalent ACT score is $21.5 + 1.7788 \times 5.4 \doteq 31.1$. (Of course, ACT scores are reported as whole numbers, so this would presumably be a score of 31.)

**1.136.** Maria's score standardizes to $z = \frac{2090 - 1509}{321} \doteq 1.81$, for which Table A gives 0.9649. Her score is the 96.5 percentile.

**1.138.** 1920 and above: The top 10% corresponds to a standard score of $z = 1.2816$, which in turn corresponds to a score of $1509 + 1.2816 \times 321 \doteq 1920$ on the SAT.

**1.142.** For a Normal distribution with mean 55 mg/dl and standard deviation 15.5 mg/dl: **(a)** 40 mg/dl standardizes to $z = \frac{40 - 55}{15.5} \doteq -0.9677$. Using Table A, 16.60% of women fall below this level (software: 16.66%). **(b)** 60 mg/dl standardizes to $z = \frac{60 - 55}{15.5} \doteq 0.3226$. Using Table A, 37.45**(c)** Subtract the answers from (a) and (b) from 100%: Table A gives 45.95% (software: 45.99%), so about 46% of women fall in the intermediate range.

**1.145.** **(a)** About 5.2%: $x < 240$ corresponds to $z < -1.625$. Table A gives 5.16% for $-1.63$ and 5.26% for $-1.62$. Software (or averaging the two table values) gives 5.21%. **(b)** About 54.7%: $240 < x < 270$ corresponds to $-1.625 < z < 0.25$. The area to the left of 0.25 is 0.5987; subtracting the answer from part (a) leaves about 54.7%. **(c)** About 279 days or longer: Searching Table A for 0.80 leads to $z > 0.84$, which corresponds to $x > 266 + 0.84(16) = 279.44$. (Using the software value $z > 0.8416$ gives $x > 279.47$.)