WS Thurs (Pg1)

Stat 202 2015 ~~3D WII~~ Fri

New and Review

Doing Tests of significance in StatCrunch

We are going to cover 3 categories of tests of significance

Stat → { Z Stats
T Stats
Proportion Stats }

We've covered → One sample

The options are { With Data
With Summary }

With Data asks for a column of numbers – the observations

With Summary asks for Summary Stats

Sample mean $\bar{x}$
Standard dev σ
Sample size n

"With data" option requires σ as well, say its optional but I don't think so
Derives $\bar{x}$ and n from collumn of numbers

Both ask for

○ Hypothesis test for μ
(test of significance involving mean of population)

Null hypothesis $H_0 : \mu = \boxed{\mu_0}$ — enter value for hypothesized mean (0 for default)

Alternative hypothesis H_a $\mu \neq$, $<$, $>$ } value above — one sided $>$, $<$; two sided $\neq$

○ $\underbrace{\text{Confidence interval}}$
we haven't covered this yet but we will

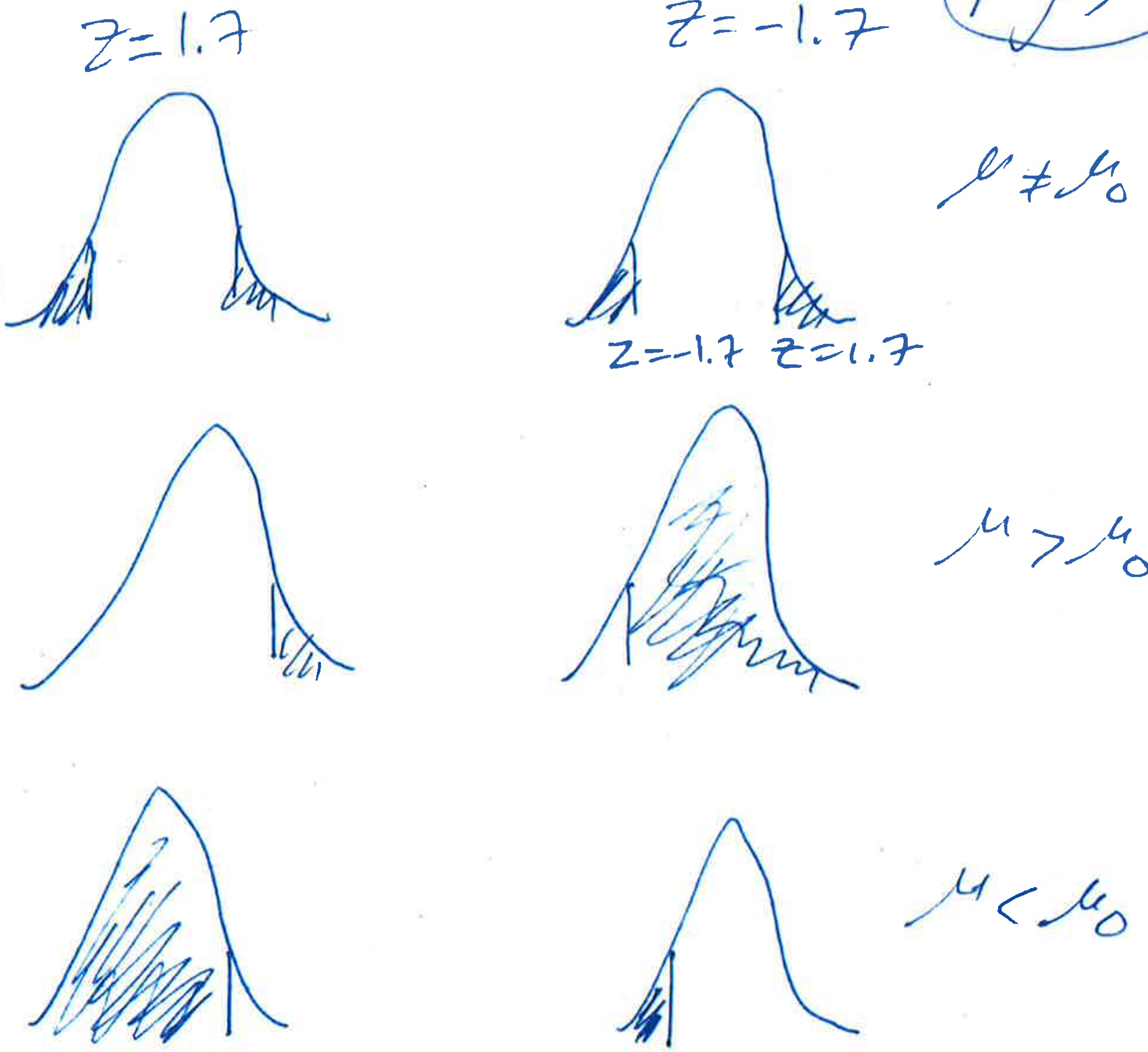
The test statistic for this test is

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

The distribution of the test statistic is $N(0,1)$

The p-value is the area under the bell curve $N(0,1)$ ~~[illegible]~~ and over values which are as extreme or more extreme as the test statistic Z.

One sided versus two sided alternatives



Significance - decide upon α ahead of time - α is level of significance traditionally $\alpha = 0.05$

If $p \leq \alpha$ significant at level α
$p > \alpha$ insignificant at level α

A significant result means that the probability of seeing results as extreme or more extreme that what is actually observed in the data ~~are sufficently small~~ assuming the null hypothesis is true is sufficiently small to reject the statement that the null hypothisis is true.

How small is that probability? that is given by the p-value!

Stat 202-2015S-W12-~~Tues~~ Wed

Review

Remember to go over In Class Exercise

Confidence Intervals

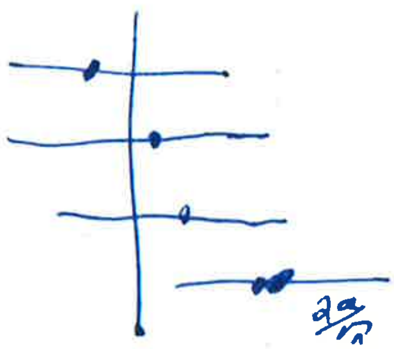
From the sample we can't infer the exact value of the parameter.

But: If certain assumptions hold we can "locate" the parameter within an interval (called a confidence interval)

Our interval won't always be "right" (won't contain the true value of the parameter) every time.

But we can control the percentage of times the interval is right. This is called the confidance level: traditionally 75%

A 95% confidence interval means that if you repeat the experiment with new data many times, each time generating a new 95% confidence interval from data then 95% of the time the parameter will be inside the interval



true value

intervals vary with data
center of interval is $\bar{x}$ for that data.
~~width is $4\sigma/\sqrt{n}$ by 68-95-99.7 Rule~~

maybe talk about this later

see next page

Each time we collect data we get a new confidence interval

Once the confidence interval is chosen, it either does or doesn't contain the parameter

So if confidence interval is (3, 4) you don't say $P(3 \le \mu \le 4) = .95$

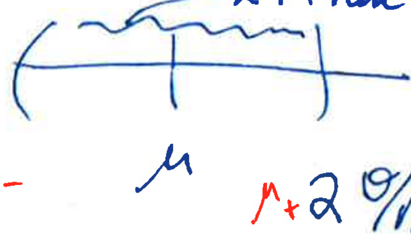
You say you are 95% confident $3 \le \mu \le 4$

$2\frac{1}{2}$

By the 68-95-99.7 Rule

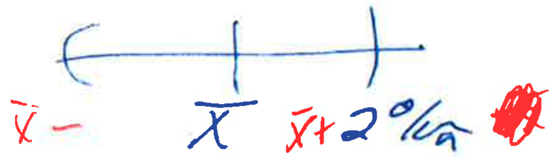
The mean falls within 2 standard deviations from μ 95% of the time

$\bar{x}$ in here 95% of the time



$\bar{X} \sim N(\mu, \sigma/\sqrt{n})$

Thus μ



both cases width is $4\sigma/\sqrt{n}$
margin of error is $2\sigma/\sqrt{n}$

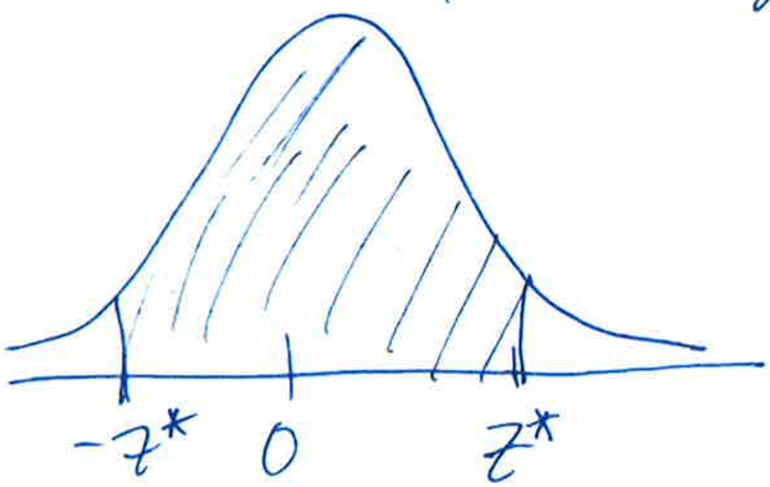
At 95% confidence level:

Confidence intervals have form

$$\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}} \qquad \bar{X} \pm m$$

estimate $\pm$ margin of error

This confidence interval is based on the z-test (assumes you know σ)



z^* chosen so that area equals confidence level (ie .95 for 95%)

$\bar{X}$ is within 1 margin of error from μ a fraction C of the time

Thus μ is within 1 margin of error from $\bar{X}$ same fraction of time.

P94

StatCrunch

Stat 202-2015S-W12-Friday

Confidence Intervals

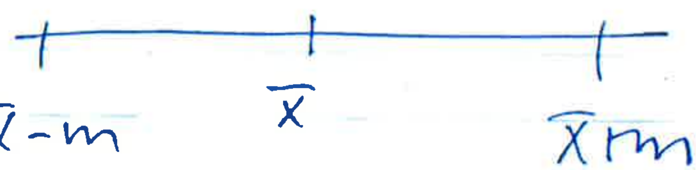
From a sample,
We can't infer the population mean μ.
The best we can do is $\bar{x}$

But we can write down an interval
which contains μ 95% of the time

The interval has the form

$$\bar{x} \pm m$$

$\bar{x}$ ← estimate; m ← margin of error



$m = z^* \frac{\sigma}{\sqrt{n}}$ where z^* is about 2 for 95% confidence intervals

If you use software instead of 68-95-99.7 Rule you'll find
$z^* = 1.960$ for 95% confidence intervals
≈ 2

by assumption

Lets say $\bar{x} = 10$, $\sigma = 2$, $n = 16$
Find 95% confidence interval, use $z^* = 2$

$$\bar{x} \pm z^* \sigma/\sqrt{n} = 10 \pm 2 \cdot 2/4$$
$$= 10 \pm 1$$
$$= (9, 11)$$

what is margin of error? 1

Let's say we draw another sample with $n=16$, we have new $\bar{x}$ 10.5, $\sigma = 2$ (same assumption) $n = 16$

$$\bar{x} \pm z^* \sigma/\sqrt{n}$$

$$10.5 \pm 2 \cdot 2/\sqrt{16} = (9.5, 11.5)$$

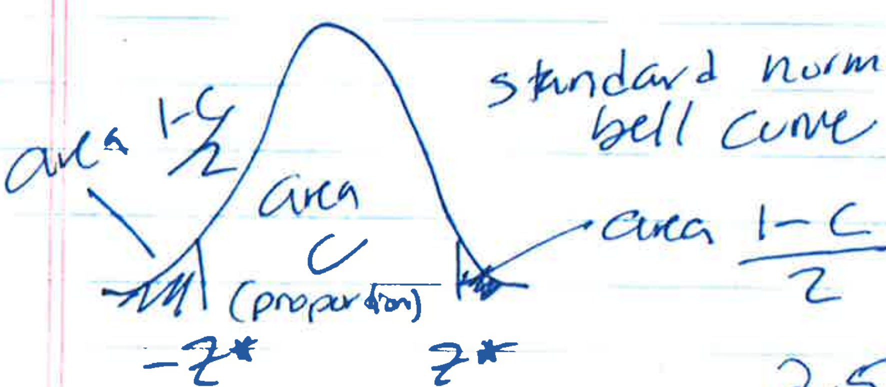
What is margin of error 1

It is the same for all samples of size 16 under our assumption ~~of~~ $\sigma = 2$ and using 95% confidence intervals so $z = 2$.

Every time we draw a new sample we are going to get a new $\bar{x}$ thus a new interval. But we are guaranteed that in a large number of samples each with $\bar{x}$ ~~that~~ 95% will contain the value of parameter μ.

Confidence intervals are traditionally 95% but other confidence levels can be considered

Confidence level	z^*
68%	1
95%	2
99.7%	3



Here C is expressed as a proportion

$95\% = .95$

$2.5\% = 0.025$

You can find z with StatCrunch

$z^* = 1.95994$

Suppose that you calculate a margin of error and decide that it is too large

$$m = z^* \sigma/\sqrt{n}$$

what can you do

- Use a lower level of confidence
  z^* will be smaller for lower confidence

- $\boxed{\text{Reduce } \sigma}$
- increase n sample size (best option)

+ Increasing n by factor of 4 decreases m by factor of 2

+ Increasing n by factor of 100 decreases m by factor of 10

What sample size do you need to have a specified margin of error.

$m = z^* \sigma / \sqrt{n}$

Solve for n

$$n = \left(\frac{z^* \sigma}{m} \right)^2$$